

# ASTRO-NATIVITY

AN ASTRONOMICAL COMPENDIUM FOR  
ASTROLOGERS.



L. NARAYANA RAO, M. A.

WITH A FOREWORD BY

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(Jyotish Mahopadhyay)



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# P R E F A C E

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Few subjects have been steeped in utter darkness and greater doubt is Indian Astrology which is to day eclipsed in its very antiquity Astrology with its sister science Astronomy went hand in hand as early as 2780 B C in India according to Mr Weber but recent astronomers like Cassini, Bailey, Gatlif and Playfair maintain there are Hindu observations which must have been made more than 3000 years before Christ and which evince even then a very high degree of astronomical science

Astrology is something like applied mathematics if Astronomy were to be considered as pure Mathematics Both of these come under one group and as one of the *Shadangas* of the Vedas

शीक्षाव्याकरण छंद निरुक्तज्योतिष तथा ।

फलपञ्चैतेषाङ्गानि वेदस्यार्हुर्मनीषिण ॥

Considerable controversy over the origin of Astronomy and Astrology in India was made attributing the science to the Greek or Arabian origin but these are amply refuted in the light of more recent researches and finding Prof Wilson and Mr Elphinstone have stated that the Hindu system is found to be their own peculiar in their method and founded on principles with which no other ancient people were acquainted and showed a knowledge of discoveries not even made in Europe till the last two centuries To cite an example is the Precession of the equinoxes Another is the diurnal rotation of the earth about its polar axis these have been discussed in Vedas in the fifth century B C and which were not known to the European astronomers even till so late as the Copernician system overpowered the Ptolemaic one which was the only system prevalent then

In passing through the various stages in the developement of the science the earliest period may be reckoned to extend from 400 B C to 2500 B C This may be termed as the "Orion" or "Vedic Sutra Period" The Aryans were still on the move At every stage they had to face the uncertainties of the lands and climes Stars were their only familiar friends and that led them to have a forecast of their future course Even on settling themselves they had many a difficulty to face—warring against aboriginal tribes clearing of forests hunting down wild animals and settling internecine feuds In all these things they turned to the stars for auspicious moments

It was thus necessary for them to calculate the positions of Sun, Moon and other planets and they had developed ere long the science to such an extent that later on, during the times of flourishing of the Persian and Babylonian kingdoms, people of India were called for and treated honourably in appreciation of the progress in every branch of learning and in the science of the future—Astrology and Astronomy—to boot.

Further later, during the times of Vikramaditya, two such eminent people were Arya Bhatta and Varahamihira who have done really good service in the field of Astronomy and Astrology.

सिध्दांतपंचकविधावपि दृष्टिवरद्व मौढ्योपरागमुखखेचरचारकन्दौ ।  
सूरसूत्रं कुसुमपुर्यभवन कलौतुभूगोळवित्कुलापार्यभटाभिधानः ॥

Then the study of the sciences was encouraged by the ruling rajahs and it attained a very high prominence. Some archaeological preserves of ancient Indian observatories are still preserved and it behoves a very high standard of advancement in that branch of study. Records of such rare and peculiar Astronomical phenomenon were made.

“शाकेन्द्र्यकीर्ति तुल्ये (१४४३) वृषदशदिमयौमासित्राणंदुनाडी ।  
तुल्ये दर्शेऽधिप्ये दिनकरदिवसे भानुतर्द्वप्रशोभूत् ॥  
तस्मिन् सर्गप्रहेऽस्तंगतमपि सकलं काव्यसप्तर्षिमुग्धया ।  
स्तारा दृष्ट्यांयकाराकलितमिह जगत्तु हाहा चकर ॥”

At present the science is almost extinct in India, quite unworthy the heritage it had. Out of sloth and indifference to the science and availability of a ready foreign ephemeris at hand and for want of encouragement at the hands of the ruling classes this has been neglected by all, with the exception of the few professional almanac-makers, who still carry on with their editing of the almanac on the old methods.

In contrast their more fortunate brethren on the Continent have been able to develop the science to a new standard with the progress of the other sciences. It is high time that we also make ourselves up to date with the information available from their researches.

It is not anti-patriotic or irreligious to accept, embrace and follow that which has been proved to be true to observation. No proof of accuracy of the methods is required as far as Astronomy is concerned except that observations

अप्रत्यक्षाणि शास्त्राणि विवादस्तेषु केवलं ।

प्रत्यक्षं ज्योतिषं शास्त्रं चद्रार्कौ यत्र शास्त्रिणौ ॥

The systems have to be rectified from time to time by bringing them nearer and nearer to observation by empirical corrections called "बीजसंस्कारः."

॥ - + - - - ग्रहणादौ सजीजकौ ॥

अपरच

ग्रहणेग्रहयोगेच कालभाह्नप्रसाधने ।

शृंगोन्नत्युदयास्तेषु ग्रहे बीजं विधीयते ॥

It is seen that the people have of late felt the need for a text book of Astronomy for calculating the planetary positions accurately and independently of any ephemeris and the present book is placed before the learned public for the benefit of the humanity for acceptance with the best of spirits

No pains have been spared in making the book self supporting for the requirements of working. A chapter on Rectification of birth time has been added which is a very useful information to the public to enable them to find out the correct birth time. The procedure indicated is to find out the time difference and thence to rectify the other houses and the planets. But in practice it will be better to find out the ascendant and Moon first to find out the rectified ascendant and the time difference and thence with the rectified time to calculate the other planets.

Before concluding I have to note a few lines in appreciation of the quick and neat Printing work executed by the Proprietor Mr K V Achuthan Nair of the Norman Printing Bureau Calicut and of the useful and suggestive criticism of my friend—to mention but a few—Messrs M R Bhat of Mangalore S J Prabhu of Calicut and D Sundar Rao Astrological Bureau Calicut to whom I offer my sincere thanks in return,

It is regretted that in spite of special efforts some corrigenda has been found necessary and followers of the book are earnestly requested to please have the corrections made before using the book.

Calicut }  
15th May 1936 }

L NARAYANA RAO

—The Author

# FOREWORD.

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Essentially a book such as Mr L. Narayan Rao M A's 'Astro-Nativity-An Astronomical compendium for Astrologers,' needs no Foreword Yet in view of the rather revolutionary doctrines promulgated in the original text on the subject, a word or two from one who has seen them put in practice may be of some value Every man is reputed to be enthusiastic over his own discoveries and corroboration from outside is correspondingly reassuring

I have for over 20 years practising and have seen in operation the greater part of the practical Astronomical Compendium contained in this volume It is therefore just to say that the methods (advocated in this self-contained book with the idea of Plane and Spherical Trigonometry as well as Tables of Trigonometrical Sines Cosines and Tangents) attain in practice, in most cases especially, a measure of success not achieved by any other system known to me

Mr Rao would, I am sure be the last to claim finality for the theoretical part of the work, but it has the merit that it offers a rational basis by its special feature of comparative old and new methods of calculation with well thought out multipliers and corrections wherever necessary

A chapter on 'Rectification of birth time' has been added at the end—a very original research different from the 'Per-Natal conceptional Epoch'—This is thoroughly Indian for many phases of this as outlined in the work have been shown to be dependent for their existence on Varaha Mihir's Brihajjataka The inferences drawn in the text may be fairly drawn and are not merely stabs in the dark In arriving at the conclusions which have been presented the greatest care has been given to the study of the cores of cases which have been analysed and critically examined before these conclusions were accepted Most of the results have been tested with a view to establishing definitely the *modus operandi* of cause and effect, and the inferences and opinions expressed are therefore true to experience and if it is desired to test them out I would earnestly request that very close attention to detail should be demanded of the experimenter before condemning the observations here put forth

This work is an attempt to bring more light to bear on a subject which bristles with difficulties and one which requires an expenditure of

much time and patience to unravel the problems associated with it. It is the fruit of years of endeavour spent in studying methods and correlating, as well as co-ordinating, the sequence and significance of the results obtained and these results have not been isolated cases, but have been repeated so frequently that in some phases of the problem they have never failed to appear when the premises of the position have been properly established.

It is not intended to give the impression that the author claims perfection for this work. He is only too well aware of the difficult ground he has to traverse.

He is entitled to the respect and encouragement from all thinking people so inclined and I am sure that the book deserves to find an important place in *Vade Mecum* of the All Astrologers Eastern as well as Western.

M. R. BHAT,

Jyotish Mahopadhyay and  
Sudhakar Ayurvedh  
Mahopadhyay

Mangalore, }  
9-5-'36 }



# ASTRO-NATIVITY.

## CONTENTS.

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			Page.
Introductory.			
Chapter	I	Plane Trigonometry	2
Chapter	II	Spherical Trigonometry	13
Chapter	III	Celestial Co-ordinates	19
Chapter	IV	Terrestrial Co-ordinates	23
Chapter	V	The Sun—Its apparent path	24
Chapter	VI	Planetary laws—The Sun—Epoch—Position of Sun at any instant and connected problem	26
Chapter	VII	Tropical longitude	53
Chapter	VIII	Time—The various kinds—Equation of Time	56
Chapter	IX	Position of the observer—Correction due to	62
Chapter	X	Position of the Ecliptic	72
Chapter	XI	Moon	89
Chapter	XII	Planets	122
Chapter	XIII	Mars	129
Chapter	XIV	Mercury	138
Chapter	XV	Jupiter	147
Chapter	XVI	Venus	157
Chapter	XVII	Saturn	167
Chapter	XVIII	Uranus	176
Chapter	XIX	Neptune	183
Chapter	XX	Rectification of birth time	189
Chapter	XXI	Transformation of Co-ordinates	201
Chapter	XXII	Hindu method of planetary Longitudes	201
Chapter	XXIII	Appendix	212

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170	6	For 481' read 48'
176 to 178		For "Uranus" read "Uranus"
177	—	Against 500000 item read signs '3' for "8",
189	6	For '1 in denominator read "1"
190	27	For "nark" read "mark"
195	16, 18	For $\frac{5x}{6}$ read $\frac{23x}{30}$ and $x = \frac{6481}{16560}$
	19	For 21'—36" as the value of $x$ , read 28'—29"
	20	For 19°—30'—7" read 19°—28'—14"
196	—	In tables IV & X houses read sec 9, V & XI houses secs 14
196	—	For "HOMSE" read "HOUSE"
197	—	In tables IV & X houses read sec 44, V & XI houses read secs 55
198	—	Read Mercury 3—23—48—26, vel 71—10 and at end of page 3—23—49—2
199	15	For $\frac{\sqrt{}}{2}$ against $\cos 30^\circ =$ , read $\frac{\sqrt{3}}{2}$
207	26	"csat" read "crist"

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(It is requested that these corrections may please be made before using this book)



## ॥ श्री ॥

इंदीवरदलश्यामं इंदिरानंदकंदनं ।

वंदारुजनमंदारं वंदेहम यदुत्तमं ॥ १ ॥

गणेशभारतीतत्त्वा आदित्यादिनवप्रहान् ।

प्रणम्यचगुरुं भक्त्या प्रहाणां गणितं ब्रुवे ॥ २ ॥

If an observer on a clear night were to look at the heavenly vault over his head, he will find countless stars, planets the Moon and such other heavenly bodies, rising at different times in the eastern horizon and setting in the western horizon, after performing their ceaseless journey. A feeling will be created in his mind whether they are definite in their heavenly path, and whether it may be possible to bring their motion under a systematic calculation, as regards their paths times of rising and setting, appearing sometimes luminous, sometimes totally disappeared and at other times some presenting phases etc.

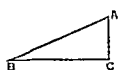
The aim and object of this book is to show how these heavenly bodies can be brought under a regular and systematic calculation. To have a really mathematical treatment of the subject, a thorough knowledge of Trigonometry—Plane and Spherical—is quite indispensable, with the help of which the planetary positions have to be calculated. Application of Logarithms will further facilitate matters but they may not be in the easy reach of all, who may as well work without logarithms. It may be found that during the course of the working of the example chosen logarithm has been applied to facilitate calculation. This may not be considered as a serious block in following this book, for the same results should be otherwise got by regular methods of multiplication and division.

The Hindu methods of calculation have also been given side by side, suggesting rectifications by way of improvement on them and it may be noted with pleasure that the direct knowledge and use of trigonometry has not been employed so much by contrivances called ज्यापदकानि (Jyapadakani), which give results within a reasonable margin of difference and standard of accuracy.

# Chapter I.

## PLANE TRIGONOMETRY.

Trigonometry as the name itself suggests is a branch of Mathematics which deals with the computation of triangles. Triangles are either plane or spherical their computations falling under Plane Trigonometry or Spherical Trigonometry respectively.



Let ABC be a rt  $\angle$ d triangle with  $\angle C$  as the rt angle. Then the ratios  $\frac{AC}{AB}$ ,  $\frac{BC}{AB}$  and  $\frac{AC}{BC}$  are called respectively the Sine Cosine and Tangent of the angle B. So also the ratios  $\frac{BC}{AB}$ ,  $\frac{AC}{AB}$  and  $\frac{BC}{AC}$  will be the sine cosine and tangent of the angle A.

Therefore in general

$$\text{Sine of an angle} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\text{Cosine of the angle} = \frac{\text{Base}}{\text{Hypotenuse}} \text{ and}$$

$$\text{Tangent of the angle} = \frac{\text{Perpendicular}}{\text{Base}}$$

For AC, BC and AB are respectively the perpendicular, base and hypotenuse with respect to angle B and BC, AC and AB become the respective parts with respect to angle A, the hypotenuse AB being the same in both the cases whether  $\angle B$  or  $\angle A$  is taken as the angle of reference.

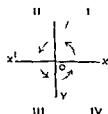
Now as defined above  $\text{sine } B = \frac{AC}{AB}$  but  $\frac{AC}{AB} = \text{cosine of angle } A$

for AC which is the  $\perp$ ar with reference to  $\angle B$  becomes the base with reference to  $\angle A$  hence what was the sine with respect to  $\angle B$  becomes the cosine with respect to  $\angle A$ . But as the  $\triangle ABC$  is a rt  $\angle$ d one the angles A and B are together equal to a rt  $\angle$  or complementary to each other.

Thus the sine of a given angle is equal to the cosine of its complement or conversely if it be required to find out the cosine of a given angle it will be enough if we find out the sine of the complement of the

angle given. This truth has been taken great advantage of for a single table of sines of all angles between  $0^\circ$  to  $90^\circ$  will serve to find out their cosines also.

The words *sine* and *cosine* are used in their shortened forms—*sine* and *cos* and these abbreviated forms only will be used hereafter. In Hindi Astronomy the terms भुज (Bhujam) and कोटि (Koti) are respectively used for them.

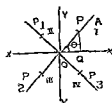


Let the lines  $XOX'$  and  $YOY'$  be drawn intersecting each other at right  $\angle$ s at the point  $O$ . Thus four quadrants are got each containing a rt  $\angle$ .

The quadrants  $XOY$ ,  $YOX'$ ,  $X'OY'$  and  $Y'OX$  are respectively called the I, II, III and IV quadrants.

It should be observed that in the numbering of the quadrants the order is not clockwise but counterclockwise which is the positive direction. It will be shown in a later chapter that it is only in this direction that the earth revolves about its axis.

All distances measured from  $O$  along  $OX$  towards the right hand side are considered positive and those along  $OX'$  towards the left negative. Similarly distances measured upwards along  $OY$  (i.e.) above the line  $XOX'$  are positive and those measured downwards (i.e.) below  $XOX'$  negative. These have to be understood very clearly as the assigning of the proper signs to the Trigonometrical ratios are based upon this fundamental conception.



Let  $XOX'$  and  $YOY'$  be the axes of coordinates with the four quadrants formed. Let  $OA$  be any line revolving about the point  $O$  in the plane of the lines  $XOX'$  and  $YOY'$ .

In the course of its revolving about  $O$  the line  $OA$  will assume different positions in the different quadrants. Let  $OP$  be a fixed length cut off the revolving line such that the distance of  $P$  from  $O$  will be the same always irrespective of the position of the revolving line.

Before the revolving line begins its revolution it would have coincided with the initial line  $OX$  and let the position  $OA$  be obtained

After describing an angle of magnitude  $\theta$  (pronounced as *thet'n*) degrees is the point whose distance from O is fixed and given

From P draw a  $\perp$ ar PQ to OX meeting OX at Q By definition

$$\left. \begin{aligned} \sin \angle POQ &= \sin \theta = \frac{PQ}{OP} \\ \cos \angle POQ &= \cos \theta = \frac{OQ}{OP} \\ \text{and } \tan \angle POQ &= \tan \theta = \frac{PQ}{OQ} \end{aligned} \right\} \begin{array}{l} \text{for PQ is the } \perp \text{ar} \\ \text{and OQ the base} \\ \text{in the } \triangle POQ \end{array}$$

When the revolving line coincided with the initial line the point P would have been on OX itself such that the point P and Q the foot of the  $\perp$ ar from P to OX would have been the same OQ would have been equal to OP itself and the angle between the revolving line and initial line also zero

Thus we would have

$$\begin{aligned} \sin 0 &= \frac{PQ}{OP} = \frac{\text{Zero}}{OP} = 0 \\ \cos 0 &= \frac{OQ}{OP} = \frac{OP}{OP} = 1 \\ \text{and } \tan 0 &= \frac{PQ}{OQ} = \frac{0}{OP} = 0 \end{aligned}$$

Next let us consider what happens to these when the revolving line assumes a position along OY Then we would have PQ becoming PO Q coinciding with O and hence QO becoming zero for YO is  $\perp$ ar to OX P is a point on OY and PQ also  $\perp$ ar to OX (i.e.)  $\angle POQ$  becomes

a rt.  $\angle$  We should therefore have  $\sin 90 = \frac{PO}{PO} = 1$ ,  $\cos 90 = \frac{OQ (= 0)}{OP}$   
 $= \text{zero}$  and  $\tan 90 = \frac{PO}{\text{zero}} = \infty$  The symbol  $\infty$  is always used for *Infinity*

which is still bigger than the biggest number we could possibly think of For zero is a number which is less than any positive quantity forgetting for a while the existence of a negative class of numbers Hence any finite number when divided by the smallest conceivable number should necessarily be the greatest number which is called *Infinity*

Thus in the I quadrant the sine ratio increases gradually from 0 to 1 the cosine ratio decreases gradually from 1 to 0 and the tangent ratio increases from 0 to  $\infty$  but not so gradually as the sine does

## BEHAVIOUR IN THE II QUADRANT.

Let the revolving line now assume any position in the II quadrant say  $OP_1$ , such that the angle  $YOP_1$  is equal to the  $\angle \theta$

Then the angle  $XOP_1 = 90^\circ + \theta$  and angle  $X'OP_1 = 180^\circ - (90^\circ + \theta)$   
for  $\angle XOP_1 + \angle X'OP_1 = 180^\circ$

Therefore the sine of  $\angle XOP_1$  is the same as the sine of  $\angle X'OP_1$  (i.e.) sine of the supplement of  $\angle XOP_1$ . Generalising the sine of an angle in the II quadrant (i.e.) when more than  $90^\circ$  but less than  $180^\circ$  will be the sine of its supplement

Further as the sine ratio involves only the  $\perp$ ar and the hypotenuse, it will be only positive as the  $\perp$ ar from the fixed point  $P_1$  to the line  $XOX'$  is still above it

As the revolving line — Radius Vector as it is called — moves on and on there will be a position, when it will lie along the line  $OX$ , in which case the  $\perp$ ar from  $P_1$  will be  $P_1$  itself. In other words, the  $\perp$ ar from  $P_1$  to the  $X$  axis becomes Zero

Now as the revolving line has exactly finished describing the first two quadrants it has described  $180^\circ$ . Therefore,  $\sin 180^\circ$  becomes Zero as also its tangent as both of them involve the  $\perp$ ar which is Zero in the present case

$$\begin{aligned}\text{Cosine of } \angle XOP_1 &= \cos \text{ of } \angle X'OP_1 = \cos (180^\circ - \angle XOP_1) \\ &= \text{cosine of supplement of the given angle}\end{aligned}$$

But as the cosine ratios involve the base and as the distances towards the left of  $O$ , in the direction  $OX$  are considered negative, as already explained, they are negative in the II quadrant, until at last when  $OP_1$  falls along  $OX$ , the base (i.e.) the distance of  $O$  from the foot of the  $\perp$ ar from  $P_1$  on  $OX'$ , becomes equal to  $OP_1$ , the cosine ratio becomes  $-1$

Therefore in the II quadrant sine ratio decreases from 1 to 0, cosine ratio decreases from 0 to  $-1$  and tangent ratio increases from  $-\infty$  to 0

## BEHAVIOUR IN THE III QUADRANT.

Let the radius vector proceed on and occupy a position in the third quadrant say  $\theta^\circ$  from the base line  $XOX'$  such that  $\angle P_2OX = \theta^\circ$

Then the ratios of the  $\angle XOP_2$  will be the same as those of  $\angle XOP_2$ , but only that the  $\perp$ ar from  $P_2$  to the base line will be below the line  $XOX'$  and hence negative. The distance of  $O$  and the foot of the  $\perp$ ar from  $P_2$  on  $OX$  will be negative also for the foot of the  $\perp$ ar will fall towards the left of  $O$ .

As the radius vector approaches  $OY$  more and more the  $\perp$ ar distance from  $P_2$  to the base line will be gradually increasing and the foot of the  $\perp$ ar approaching  $O$ , until at last when  $OP_2$  falls along  $OY$   $OP_2$  will be the  $\perp$ ar itself and the foot of the  $\perp$ ar will coincide with  $O$ .

That is the sine ratio will be numerically increasing from 0 to 1 and the cosine ratio numerically decreasing from 1 to 0 though both will be negative algebraically.

The tangent ratio as it involves both the  $\perp$ ar and the base which are both negative in this quadrant will be positive and increasing from Zero to positive infinity.

Thus the ratios of an angle in the III quadrant are numerically the same as those of the given angle less  $180^\circ$  for as in the example taken the ratios of  $\angle XOP_2$  are the same as those of  $\angle X'OP_2$ . Care should be taken to put the proper sign before each of the ratios so got.

## BEHAVIOUR IN THE IV QUADRANT.

Lastly let the radius vector occupy a position say  $OP_3$  in the IV quadrant. The angle traced by the radius vector from its initial position along  $OX$ , is  $\angle XOP_3$  measured in the positive or counter clock wise direction.

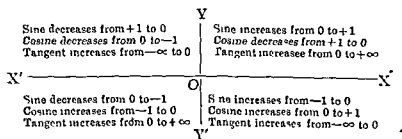
The trigonometrical ratios of the  $\angle XOP_3$  will be the same as those of  $\angle P_3OX$ , for the  $\perp$ ar from  $P_3$  to the initial line  $OX$  and its foot will be the same for both the  $\angle$ s.

Also the sine ratio will be negative in this quadrant as the  $\perp$ ar from the point  $P_3$  to the initial line still continue to be below it the cosine ratio will be positive as the foot of the  $\perp$ ar has once again fallen to the right of  $O$  and towards  $OX$  and the tangent ratio will be negative as the  $\perp$ ar is negative while the base is positive

At the end of the third quadrant the sine cosine and tangent have respectively attained the values  $-1$ ,  $0$  and  $+\infty$ . Now in the fourth quadrant the sine increases from  $-1$  to  $0$ , cosine from  $0$  to  $+1$  and tangent from  $-\infty$  to  $0$ .

It may not be clear for new learners of Trigonometry how the tangent ratio at the ends of the first and third quadrants though approaching  $+\infty$  could suddenly begin to increase from  $-\infty$  to  $0$  as the angle just tends to enter the II and IV quadrants. For the tangent is as primarily defined a ratio of the  $\perp$ ar and the base. The  $\perp$ ar is positive in the I and II quadrants and the base positive in the I and IV quadrants such that the tangents are positive only in the I and III quadrants. The tangent ratio which attains the value  $+\infty$  at  $90^\circ$  suddenly turns to  $-\infty$  by dint of the base assuming the negative sign as the angle traced by the radius vector just comes into the II quadrant as the tangent ratio involves the base in its denominator. So also for the sudden change from  $+\infty$  to  $-\infty$ , as the angle just passes from the III to the IV quadrant.

The following diagram gives us the scheme for remembering the signs and natures of the ratios



Or

	Sine	Cosine	Tangent
I quadrant	+	+	+
II quadrant	+	—	—
III quadrant	—	—	+
IV quadrant	—	+	—

It could have been seen by this time that the tangent of an angle, which has been defined as the ratio of the  $\perp$ ar to the base could be also defined as the ratio of the sine to its cosine,

$$\text{for, tangent} = \frac{\perp\text{ar}}{\text{base}} = \frac{\perp\text{ar}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{base}} = \frac{\perp\text{ar}/\text{hypotenuse}}{\text{base}/\text{hypotenuse}} = \frac{\text{sine}}{\text{cosine}}$$

## TO FIND THE SINE OF ANY ANGLE.

First of all find out the quadrant wherein the given angle lies (i.e.) whether less than  $90^\circ$  or greater than  $90^\circ$  but less than  $180^\circ$  or greater than  $180^\circ$  but less than  $270^\circ$  or lastly, greater than  $270^\circ$  and less than  $360^\circ$

When in the I quadrant, the angle itself will be the argument for finding out the sine, when in the II subtract it from  $180^\circ$ , the remainder will be the sine argument in the III quadrant the excess over  $180$  will be the required sine argument, and lastly in the IV quadrant the defect of the given angle from  $360^\circ$  will be the sine argument. With the argument having got the numerical value of the sine from the tables annexed, the proper sign according to the quadrant the given angle lies in will have to be pre-fixed to it.

## TO FIND THE COSINE OF THE SAME.

After having got the sine argument subtract the sine argument from  $90^\circ$  always and the sine of this defect will be the cosine of the angle given. Care should be taken to prefix the proper sign according to the quadrant occupied.

## TO FIND THE TANGENT OF THE SAME.

After having found out the sine and cosine as told above with the proper sign divide the sine by the cosine. The quotient will be the tangent with the proper sign.

There are other three ratios viz cosecant secant and cotangent which are the reciprocals of the sine cosine and tangent respectively. They can be as well got rid of by employing  $\frac{1}{\sin}$ ,  $\frac{1}{\cos}$ , and  $\frac{1}{\tan}$  respectively. There are two more—versine and coversine which stand for  $(1-\cosine)$  and  $(1-\text{sine})$ , but they occur very seldom.



## CORRESPONDING DETAILS IN HINDU ASTRONOMY.

The measurement of angles in Sexagesimal measure is found to have been in use since times immemorial and of wide acceptance though in some places centesimal measure also has been used. The former has got decidedly more advantages than the latter and it is only the Sexagesimal measure that Hindus have used.

In the Sexagesimal measure, a degree is divided into 60 parts called minutes and each such minute being further divided into 60 seconds. These minutes and seconds are of arc, as distinguished from the more commoner ones of time.

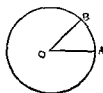
In the Hindu Astronomy the terms अंश (or भाग), कला and तिकला are employed corresponding to degrees, minutes and seconds. 30 degrees are supposed to comprise a राशि or a sign. These signs are twelve in number, for  $\frac{360}{30} = 12$ . These signs are called as follows with the corresponding equivalents:

मेघ =	Aries	तुला =	Libra
वृषभ =	Taurus	वृश्चिक =	Scorpio
मिथुन =	Gemini	धनुः =	Sagittarius
कर्क =	Cancer	मकर =	Capricorn
सिंह =	Leo	कुंभ =	Aquarius
कन्या =	Virgo	मीन =	Pisces

## THE RADIAN MEASURE (त्रिज्या)

As the necessity to convert angular measure into circular measure and vice versa, will be felt very often, a measure has been employed, called the "Radian". It may seem from its name, that it may have something in relation with the radius of a circle and in fact it does so.

A radian is defined as the angle subtended at the centre of a circle, by an arc equal to the radius.



Let a  $\odot$  centre  $O$  be drawn and let  $OA$  be any radius. Along the circumference step off a portion  $AB$  equal to the radius  $OA$ .

[N B—Care should be taken not to cut off an arc with radius equal to that of the given circle. But the portion to be stepped off should be by placing a length equal to the radius along the circumference.]

Join OB. Then  $\angle AOB$  will measure the value of a radian

For,  $\frac{\angle AOB}{360^\circ} = \frac{\text{arc AB}}{\text{circumference}}$  as the arcs are proportional to the angles subtended at the centre in same circles or equal circles by a well known geometrical truth

$\therefore \angle AOB = \frac{360 \times \text{arc AB}}{\text{circumference}}$  but arc AB—radius by our data =  $r$  and circumference =  $2\pi r$ , where  $\pi$  is the Greek letter ( $\pi$ ) whose values have been taken differently by different mathematicians viz  $3\frac{1}{4}$ ,  $3\frac{1416}{113}$  etc. The value  $\frac{113}{355}$  has been used by Bhaskaracharya the eminent Hindu Astronomer, and approaches very much to the real value which is incommensurable and still under investigation and research

Taking this value  $\frac{113}{355}$ , the No of degrees in a radian is equal to

$$\begin{aligned}\frac{360 \times r}{2\pi \times r} &= \frac{360}{2\pi} = \frac{180}{\pi} = \frac{36 \times 113}{71} \\ &= 57^\circ - 17' - 44''.79 \\ &= 57^\circ - 17' - 44'' \cdot 79 \\ &= 206265''\end{aligned}$$

To find out the भुज or the argument for finding the sine

#### HINDU METHOD

- 1 If राशि is 0 1 or 2 the same is भुज.
- 2 If राशि is 3, 4 or 5 subtract from 6 signs
- 3 If राशि is 6 7 or 8 subtract 6 signs from the given one
- 4 If राशि is 9, 10 or 11, subtract from 12 signs

#### COMPARE

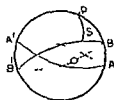
- 1 I quadrant the angle itself will be the argument
- 2 II quadrant given angle subtracted from  $180^\circ$
- 3 III quadrant subtract  $180^\circ$  from the given angle
- 4 IV quadrant given angle subtracted from  $360^\circ$

ing the distances of the planets from the earth, given their distances from the Sun, distance of Earth from Sun and the difference of the Heliocentric longitudes of the planets and Earth, which will be dealt at length in the concerned chapters

## Chapter II

### SPHERICAL TRIGONOMETRY.

The heavenly vault overhead is hemispherical in shape and concave to the observers on the earth and the positions of the stars and other heavenly bodies as seen by them are merely the projections of the bodies on the spherical vault. The lines of arc joining the positions of three bodies joined two at a time as seen by the observer form spherical triangles, the computation of which cannot be made unless with a fair working knowledge of Spherical Trigonometry



Let  $ABPA'$  be a sphere described about an observer  $O$  at the centre

Any section of the sphere, say  $AOA'$  passing thro'  $O$  - or any central section of the sphere is called a great circle, as different from sections of the sphere not passing through  $O$ , in which case they are called small circles

We shall have to deal with spherical  $\Delta$ s formed by great circles only

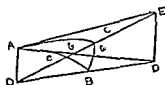
If  $BB'$  be another great circle and if  $S$  be any star and if its position be joined to  $P$  by an arc of another great  $O$  passing through  $S$ , then the triangle formed by the arcs  $PS$ ,  $SB$  and  $PB$  is a spherical triangle of the type described in previous paragraphs

It could be easily conceived that if  $OS$ ,  $OP$  and  $OB$  are joined by straight lines, they will be all equal for,  $P$ ,  $S$  and  $B$  are points on the sphere whose centre is  $O$  each being the radius of the sphere. The sides of the spherical triangle are denoted by the angles subtended at the centre of the sphere by the respective arcual sides and the angles of the spherical  $\Delta$  are

those between the tangents at the vertices to the great circles passing through the vertex and whose arcs are the sides of the spherical triangle

[N B —The tangent mentioned here is defined as a line touching a given circle at two ultimately coincident points or as the limiting position of a chord and should not be confusedly mixed up with the trigonometrical tangent which is purely a number

## RELATION BETWEEN THE SIDES AND ANGLES OF A SPHERICAL TRIANGLE.



Let ABC be a spherical triangle formed by the arcs AB BC and CA of three great Os of the sphere centre O

The sides AB BC and CA are respectively  $c$ ,  $a$  and  $b$  being the measure of the angles subtended at O the centre of the sphere by the arcs mentioned that is  $a = \text{angle subtended by arc BC at O}$

$$= \angle BOC$$

Similarly  $b = \angle AOC$  and  $c = \angle AOB$

Suppose tangents be drawn to the arcs AB and AC at the point A which is also the vertex of the spherical  $\triangle ABC$  the angle between the tangents will be the  $\angle BAC$  of the spherical triangle Thus if AD and AE be the tangents drawn at A meeting OB and OC produced at D and E respectively then  $\angle DAE = \angle BAC$  of the spherical triangle similarly the angle between the tangents at B to the two arcs AB and BC will measure the  $\angle ABC$  of the spherical  $\triangle$  so also  $\angle ACB$

OA=OB=OC being radii of the same sphere and let each be equal to  $r$ .

Since AD is a tangent drawn at A to the arc AB  $\angle OAD$  is a rt  $\angle$  (for by Geometry a tangent to a O and the radius thro the point of contact are  $\perp$ ar to each other)

$$\therefore \frac{AD}{AO} = \tan \angle AOD = \tan \angle AOB = \tan c,$$

$$(i.e.) AD = AO \tan c = r \tan c$$

$$\text{Similarly } AE = r \tan b$$

and  $\angle DAE = \angle A$  of the spherical  $\triangle$

Thus in the plane  $\triangle ADE$  AD AE and  $\angle A$  are known

$$\therefore DE^2 = AD^2 + AE^2 - 2 AD AE \cos \angle DAE$$

(refer plane Trigonometry portion for the relation of third side with two sides and included angle)

$$\begin{aligned}\therefore DE^2 &= r^2 \tan^2 c + r^2 \tan^2 b - 2 r^2 \tan b \tan c \cos A \\ &= r^2 (\tan^2 c + \tan^2 b - 2 \tan b \tan c \cos A)\end{aligned}$$

$$\text{Now } \frac{OA}{OD} = \cos \angle AOD = \cos c \quad (\because) \quad OA = OD \cos c$$

$$\therefore OD = \frac{OA}{\cos c} = \frac{r}{\cos c}$$

$$\text{Similarly } OE = \frac{r}{\cos b} \text{ and } \angle DOE = \angle BOC = \alpha$$

$$\text{but } DE^2 = OD^2 + OE^2 - 2 OD OE \cos \angle DOE$$

$$\begin{aligned}&= \frac{r^2}{\cos^2 c} + \frac{r^2}{\cos^2 b} - \frac{2r^2}{\cos b \cos c} \cos \alpha \\ &= r^2 \left[ \frac{1}{\cos^2 c} + \frac{1}{\cos^2 b} - \frac{2 \cos \alpha}{\cos b \cos c} \right]\end{aligned}$$

as these should be equal equating the expressions that are equal to  $DE^2$ , we get

$$[\tan^2 c + \tan^2 b - 2 \tan b \tan c \cos A] = \left[ \frac{1}{\cos^2 c} + \frac{1}{\cos^2 b} - \frac{2 \cos \alpha}{\cos b \cos c} \right]$$

$$\therefore \frac{\sin^2 c}{\cos^2 c} + \frac{\sin^2 b}{\cos^2 b} - \frac{2 \sin b \sin c}{\cos b \cos c} \cos A = \frac{1}{\cos^2 c} + \frac{1}{\cos^2 b} - \frac{2 \cos \alpha}{\cos b \cos c}$$

[By plane trigonometry we have  $\sin^2 \theta + \cos^2 \theta = 1$  for  $\sin \theta = \frac{\text{perp}}{\text{hypot}}$  and  $\cos \theta = \frac{\text{base}}{\text{hypot}}$

$$\therefore \sin^2 \theta + \cos^2 \theta = \frac{(\text{perp})^2 + (\text{base})^2}{(\text{hypotenuse})^2} = \frac{(\text{hypotenuse})^2}{(\text{hypotenuse})^2} = 1]$$

$$\begin{aligned}\therefore \frac{2 \sin b \sin c}{\cos b \cos c} \cos A &= \frac{1 - \sin^2 c}{\cos^2 c} + \frac{1 - \sin^2 b}{\cos^2 b} - \frac{2 \cos \alpha}{\cos b \cos c} \\ &= 1 + 1 - \frac{2 \cos \alpha}{\cos b \cos c} = 2 - \frac{2 \cos \alpha}{\cos b \cos c}\end{aligned}$$

Dividing by 2 throughout we have, by multiplying thereafter by  $\cos b \cos c$

$$-\sin b \sin c \cos A = \cos b \cos c - \cos \alpha$$

$$\therefore \cos \alpha = \cos b \cos c + \sin b \sin c \cos A$$

This is a very important formula and similar expressions for the other sides  $b$  and  $c$  can be written down symmetrically from these as follows

$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$

$$\text{and } \cos c = \cos a \cos b + \sin a \sin b \cos C$$

These three formulæ give rise to another set of three formulæ thus, we have already deduced

$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$

Substitute for  $\cos a$  in this, we get

$$\begin{aligned}\cos b &= \cos c (\cos b \cos c + \sin b \sin c \cos A) + \sin c \sin a \cos B \\ &= \cos b \cos^2 c + \sin b \sin c \cos c \cos A + \sin c \sin a \cos B \\ \therefore \cos b - \cos b \cos^2 c &= \sin c [\sin b \cos c \cos A + \sin a \cos B] \\ (1) \quad \cos b (1 - \cos^2 c) &= \cos b \sin^2 c = \sin c [\sin b \cos c \cos A \\ &\quad + \sin a \cos B]\end{aligned}$$

Dividing by  $\sin c$  throughout we have

$$\begin{aligned}\cos b \sin c &= \sin b \cos c \cos A + \sin a \cos B \\ &= \sin b (\cos c \cos A + \frac{\sin a}{\sin b} \cos B)\end{aligned}$$

$$\therefore \frac{\cos b}{\sin b} \sin c = \cos c \cos A + \frac{\sin a}{\sin b} \cos B$$

$$\begin{aligned}(1) \quad \cot b \sin c &= \cos c \cos A + \frac{\sin A}{\sin B} \cos B \\ &= \cos c \cos A + \cot B \sin A\end{aligned}$$

$$\left. \begin{aligned}\therefore \sin A \cot B + \cos c \cos A &= \cot b \sin c \\ \sin B \cot C + \cos a \cos B &= \cot c \sin a \\ \sin C \cot A + \cos b \cos C &= \cot a \sin b\end{aligned} \right\} \text{II}$$

$$\left[ \text{for } \frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \text{ in spherical trigonometry on the}$$

analogy of

$$\left[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \text{ in plane trigonometry} \right]$$

The first set of three formulæ are to be used when two sides and contained angle are known and it be reqd to find out the third side

The second set of three formula are to be used when any three adjacent parts are known and it be required to find out the fourth

These apply to all spherical  $\Delta$ s formed by three great circles But if two of these arcs meet at rt  $\angle$ s (i.e) if one angle of the spherical  $\Delta$  becomes a rt  $\angle$ , the triangle becomes a rt  $\angle$ d spherical triangle The above formulæ become much simpler and a simpler method has been found out to remember for applying the formulæ suitable to the occasion

$$\therefore \sin (90-b) = \tan (90-C) \times \tan (90-A)$$

(i.e.)  $\cos b = \cot C \cot A$  This also agrees with what has been independently arrived at

It can be illustrated how all the other sets of the Napierian analogies are proved

These will be of very great advantage in the transformation of celestial co-ordinates for example—given celestial longitude and latitude to find out the R.A. and declination and vice versa etc

A table of sines, cosines and tangents is given in the appendix and it will save the time and trouble of referring to a book of Trigonometrical tables. Mere sine ratio alone will enable us to find out the other ratios but for the convenience of the users of this book they are made ready at hand and easy of reference

## Chapter III

### CELESTIAL CO-ORDINATES.

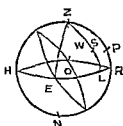
The heavenly vault appearing to all observers on the earth has already been described as a hemisphere on which the positions of heavenly bodies are projected. The data which are essential in locating the position of any heavenly body as will be apparent on the celestial sphere are called the celestial co-ordinates

The horizontal section of the celestial sphere with the observer at its centre is called the horizon and the pole of this section (i.e.) the point just above the head of the observer is called the Zenith and the corresponding point below, the Nadir. [By the word *pole* is meant that point on the sphere which when joined to any point on the circumference of the circular section of which it is the pole, measures  $90^\circ$  of arc. Or it is a point on the sphere such that it measures exactly  $90^\circ$  of arc to any point on the circumference of a central section whose pole is the point]

When the observer faces the north he will observe the pole star towards which the axis of the earth is always pointing in its diurnal rotation as well as in its annual rotation about the sun. Where this direction cuts

e celestial sphere will be the pole of the celestial equator or the celestial pole. In other words the celestial equator is that circular central section of the sphere perpendicular to the fixed direction of the pole. This celestial pole should not be confused with other—terrestrial pole.

Where the celestial equator cuts the horizon of the observer are the East and West points. That section of the celestial sphere passing through the Zenith and the East and West points and terminated by the horizon is called the prime vertical.



HOR—horizon

P—pole of the celestial equator

Z—zenith of the observer

N—The Nadir

EZW—prime vertical

E and W—being the East and West points

## CO-ORDINATES REFERRED TO ZENITH AND HORIZON.

Let the celestial sphere of the observer be drawn as in the above figure and be named with the usual notation. Let S be a star. If a great circle be drawn through Z and S terminating it with the horizon then ZSL is the vertical through the given star.

If SO be joined,  $\angle SOL$  subtended by the arc SL at the centre O of the celestial sphere where the observer is stationed or merely the arc SL is called the altitude of the star and its complement ZS the zenith distance. For, Z being the pole of the section HO+R of the sphere  $ZL$  is  $90^\circ$ , but  $ZS + SL = ZL$ . ZS is the complement of SL or the zenith distance is the complement of the altitude. HL is called the Azimuth and is measured from the south point towards the east or west when the North pole is above the horizon or from the North in the other case. Hence the azimuth and the altitude or zenith distance will determine completely the position of a heavenly body at a given instant. The zenith distance and altitude are called in Hindu Astronomy as *नलतं* and *उन्नतं* respectively.



# Chapter IV.

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## TERRESTIAL CO-ORDINATES

The different places on the earth's surface are distinguished from one another by their longitudes and latitudes

For an observer on the terrestrial equator the celestial pole will appear along his horizon and for one on the terrestrial pole the celestial pole will appear directly over head. Thus for any intermediate position the altitude of the celestial pole will be the latitude of the place

The latitude of a place is the angle subtended at the centre of the earth by an arc equal to the distance of the place from the terrestrial equator measured along the terrestrial meridian passing through the terrestrial pole and the place of the observer

In the figure of the last para of the last chapter PR will be the latitude of the place being altitude of the pole and ZP will be the co-latitude which is only an abridged form of complement of latitude

The longitude of a place is the angle made by the meridian of the place with some arbitrarily chosen meridian and is measured by the arc of the terrestrial equator intercepted between these two meridians.

The uniform motion of the celestial axis takes place in a sidereal day. It is also stated that RA and declination are to the celestial equator and longitude and latitude to the terrestrial equator.

This day

## Chapter V.

### THE SUN, ITS APPARENT PATH.

Observations show that the Sun's apparent daily path is constantly changing. The points where he rises and sets, the time he is above the horizon, the greatest height he attains all vary day by day and hence the declination of the sun also varies. Otherwise he would rise and set always at the same point and attain the same meridian altitude every day. That the apparent Sun has a progressive motion from west to east may be seen from the fact that stars seen near the horizon at sunset each successive day remains a less time visible and after a few evenings will disappear altogether or that stars in the east seen before sunrise in the eastern horizon each successive morning will be seen for a longer time and attain a greater altitude before the sun's light can overpower them.

This seeming motion of sun is only apparent due to a motion of translation (i.e.) a motion of the earth round the sun independent of the rotation of the earth on its own axis.

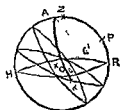
If a globe be made of some transparent material and the Sun's position at noon be marked on it by an observer as seen through the globe as also the angle his declination circle makes with that of a known star whose daily position also is marked on the globe it will be found that the different positions of the sun will arrange themselves on a great circle cutting the celestial equator in two opposite points  $\Upsilon$  and  $\varpi$  the first point of Aries and of Libra respectively and inclined to it at an angle of mean value of about  $23^{\circ}-27^{\circ}-30^{\circ}$ .

This great circle the plane of which contains the sun's apparent annual path is called the *Ecliptic* and the angle the ecliptic makes with the equator is called the obliquity of the ecliptic.

As the ecliptic is thus the circle formed by the joining of the different points of the declination circles it is called *व्रतवृत्तं* in Hindu Astronomy and the equator on which *R A* is measured the *विषुव वृत्तं*. The points  $\Gamma$  and  $\omega$  where the equator and the ecliptic cut each other are called the *equinoctial points* or points of zero declination. The two points equidistant from  $\Gamma$  and  $\omega$  either way are called the *solstitial points*.

It should be noted that due to the inclination of the ecliptic to the equator the poles of the equator and the ecliptic are not in the same plane as those of the equator and the horizon.

The variation in the lengths of day and night are due to a gradual change in the declination which is at once due to the progress of the sun's apparent position on the ecliptic.

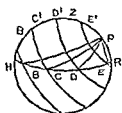


With the usual notation let the celestial sphere of the observer be drawn and let the ecliptic and equator be marked on it.

Suppose *K* is the position of the apparent sun at a given instant and let us suppose the declination of the sun to remain stationary throughout the day.

To find where the sun will appear to rise, find out the *N P D* ( $=PK$ ) of the sun and draw a  $\parallel$  of declination (marked as  $a A a'$ ) cutting the horizon at the points  $a a'$  and the meridian at *A* the point of the highest altitude. Then  $a A a'$  will be the apparent diurnal path. But due to the slight increase of declination in the course of the day arc  $a A$  will not be equal to *A a*.

As the declination and therefore the *N P D* are changing every day, the arc  $a A$  representing the semidiurnal apparent path will change and hence the change in the duration of day.



For different *NPD*'s *BP CP DP* and *EP*, we get the semidiurnal paths, *BB'*, *CC'*, *DD'* and *EE'* which can be seen to be different in their lengths which indicate the consequent difference in the lengths of the days.

The variation in the length of the days is a thing the Hindu astronomy has not lost sight of and is, in fact, one of the elements of a Hindu Astronomy. For births at night, the time after night fall is added to the day. The unity to get the time elapsed since sunrise of that day, which is the length of the day. The Hindu day is equal to the sidereal day.

This day

## Chapter VI

### PLANETARY LAWS—SUN—EPOCH—POSITION OF SUN AT ANY INSTANT AND CONNECTED PROBLEMS.

The apparent position of the sun is the first thing to be arrived at for, all other problems depend on it. The apparent position of the sun is really the position of the earth in its orbit about the sun together with  $180^\circ$ . For, an observer centred at the centre of the earth will see the sun through an angle of  $180^\circ$  more than he would be seen by a similar observer centred at the centre of the sun.

It has been enunciated by Kepler, as regards planetary motion that all planets

- 1 Describe an ellipse about the sun in one of its foci,
- 2 Move such that the radii vectores joining the sun and planets describe equal areas in equal times,
- and 3. Have the squares of their periodic times varying as the cubes of the semimajor axes

These have been later on found out to be necessary consequences of Newton's Law of Universal attraction

Therefore to find out the position of a planet at a given instant, we require the following elements of its orbit

1. Mean heliocentric longitude at a determinate instant called the Epoch
- 2 Mean angular velocity per day in its orbit
- 3 The semi-major axis of the ellipse
- 4 The eccentricity of the orbit
- 5 The heliocentric longitude of the Node with its annual velocity

- 6 The inclination of the planetary orbit to the ecliptic
- 7 The longitude of the apse of the planetary orbit with its annual velocity

By the term Heliocentric mean longitude is meant the angle at the sun formed by the radius vector—the line joining the sun and the planet to its own orbit—and a fixed direction of the first point of Aries from which all angles are reckoned

The apses are two positions separated by  $180^\circ$  at which the planet appears from the sun to have the maximum or minimum Heliocentric velocity

Of the various elements given 3 and 4 determine the magnitude of the ellipse 5 and 6 determine the position of the plane of the orbit relative to the ecliptic 7 indicates the direction of the major axis in the ellipse determined by 3 and 4 and lastly 1 and 2 determine the mean heliocentric position at a subsequent date

These with the rules of elliptical motions obtain the positions of the planets in their orbits at any subsequent instant

These elements are determined by three complete sets of observations of Right Ascension and declination from which the corresponding geocentric latitudes and longitudes are found, thus giving two equations connecting the elements of the orbit Though theoretically sufficient it is practically found to be simpler and more accurate to make observations at particular times when the planet occupies selected positions favourable for finding each element in turn

## EPOCH.

The epoch is a date fixed by each astronomer, arbitrarily for obvious reasons such as the beginning of a century or a rare Annular or total eclipse of the sun or a transit of Venus etc Thus different people have taken different epochs The foremost of the eighteen siddhantas of the Hindu Astronomy is the Surya Siddhanta and has for its epoch the beginning of the Kaliyuga which commenced in 3102 B C February The eighteen siddhantas are as follows —

सूर्यः पितामहो व्यासो वसिष्ठोऽत्रिः पराशरः ।  
 कश्यपो नारदो गरुडो मरीचिर्मनुगिराः ॥  
 लोमशः पौलिशश्चैव च्यवनो यवनो मनुः ।  
 शौनकोऽष्टादशश्चैते ज्योतिः शास्त्रप्रवर्तकाः ॥

The extent of the Kaliyuga is  $\frac{1}{10}$  of the period called Mahayuga consisting of 320,000 mean solar years. This may seem arbitrary but it is not. The unit is seen that this is the L. C. M. of the different sidereal solar axis takes place of the different planets converted to years. The called a sidereal day from the beginning of Kaliyuga to the epoch is already particular chooser of an Epoch and fixed. Then if it be

This day the number of days from the epoch we have first to find the number of days from Kaliyuga commencement to the given date and hence to subtract the former, when we get the required number of days from epoch.

## TO CALCULATE THE DAYS ELAPSED FROM THE COMMENCEMENT OF KALIYUGA OR TO FIND OUT THE अहर्णि.

The duration of Kaliyuga as already told is 4,32,000 years. This has been divided into 6 eras named after the kings who last ruled or who are supposed will rule during the Kaliyuga.

1. *Yudhishtra Era* (युधिष्ठिर शकं) The first 3044 years from the commencement of the Kaliyuga
2. *Vikrama Era* (विक्रम शकं) The next 135 years from the ending of the Yudhishtra Era
3. *Salivahana Era* (शालिवाहन शकं) The next 18000 years from the ending of the Vikrama Era
4. *Vijayabhinandana Era* (विजयाभिनेन्दन शकं) The next 10000 years from the ending of Salivahana Era
5. *Nagarjuna Era* (नागार्जुन शकं) The next 400000 years from the ending of Vijayabhinandana Era and lastly
6. *Kalki Era* (कल्की शकं) The last 821 years from the ending of Nagarjuna Era, thus making altogether 4,32,000 years

We are therefore now in the Salivahana Era and 3179 years had elapsed in the Kaliyuga when Salivahana Era began. Thus the number of Salivahana Era years elapsed together with 3179 years will give the years elapsed from the commencement of Kaliyuga.

The Salivahana Saka years are lunar and solar according as the system adopted. The Suryasiddhanta considers the lunar reckoning of the

months and adjusts it to the solar one in the former system, while in the latter system no reference is made—nor is it necessary—to the lunar reckoning at all. The length of the year according to the lunar reckoning is 354 367 days, while the solar reckoning has for its year a length of 365 258 628 7 days though according to modern astronomers it is given as 365 256 374 days.

In any case the year as per the lunar reckoning or चंद्रमान, being 11 days shorter than the solar one the adjustment is brought about by adding 1 lunar month for every 32 5 months nearly or 28 months for a period of 911 lunar months. But this takes us farther and a further correction the rate of 11 days for every 703 days will have to be subtracted to bring it in a pace with the solar sidereal year.

$$\text{For, } 30 \times 12 \times \frac{911}{11} \times \frac{703}{703} = 30 \times 12 (1 + \frac{1}{11}) (1 - \frac{1}{703}) = 365\ 258\ 628\ 72$$

This is the method of Suryasiddhanta and we shall adhere to this only for finding the number of days from the commencement of Kaliyuga. We shall not take the modern revised rate for the length of the sidereal year, as there is no loss of generality or accuracy provided we take the respective additive months to suit the particular rate taken and as both methods give the number of days elapsed since the commencement of the same epoch.

The controversy as to which rate has to be chosen has still not reached a definite conclusion among the astronomers and in fact it forms one of the important questions discussed during any astronomical conference of the kind held in India. The difference is due to a primary difference in the lengths of the tropical year. The tropical year of the Suryasiddhanta is to that of the Modern Astronomy as 359°-59'-6" to 359°-59'-9" 78 and therefore the sidereal years are in the inverse ratio of the same numbers.

The reason why the followers of the Suryasiddhanta do not embrace the more correct value of the modern people, is that the rules for additive and subtractive months as laid down in Suryasiddhanta are not fulfilled in case the present rate were to be accepted.

We shall not dwell on this controversy but content ourselves with the ancient method only as far as finding out the number of days elapsed is concerned.

The commencement of Salivahana year (Lunar Reckoning) is the day after the new moon in the lunar month in March or April and the first

nar month of the year usually commences between the 15th of March and 14th April. The first month is called Chaitram (चैत्र) and the other months are Vaisakam (वैशाख), Jyesta (ज्येष्ठ), Ashada (आषाढ), Bhādrapada (भाद्रपद), Āsvijā (आश्विन), Kārttika (कार्तिक), Pūshya (पौष), Magha (माघ) and Phalgunā (फाल्गुन).

This day with that is added to the number of past lunar months to find the space those of the solar reckoning is called the (अधिकमास) Adhikamasa (= Additive month). Each lunar month is divided into two halves—the light and the dark fortnight शुक्लपक्ष and कृष्णपक्ष respectively. Each fortnight has in general 15 days.

## TO FIND THE NUMBER OF DAYS ELAPSED FROM THE KALIYUGA EPOCH TO A GIVEN DATE OF THE HINDU LUNAR RECKONING.

To the Salivahana year add 3179 we get the number of years from the epoch of Kaliyuga. Multiply this by 12 and add to the product the number of months past from चैत्र or Chaitram. To this sum add  $\frac{1}{11}$  of itself (any final balance on division should be left off even if the balance were more than half). This is the correction for additive months and will bring it on a level with the solar reckoning. Multiply the months so corrected by 30 and add to it the number of days passed since the last new moon. Subtract therefrom  $\frac{1}{7}$  of itself—ignoring the remainder as before. This is the minus correction for क्षयतिथि or for the extra days included by reckoning 30 thithies instead of 29½ days only which is one lunation of the Moon. The remainder will give the number of days from the beginning of Kaliyuga of the present Mahayuga or will give the required अहर्गण (Aharganam).

**TEST** Divide the number of days (अहर्गण) or Aharganam by 7 and the remainder counted from Friday as 1 will give the weekday of the day chosen. But as the weekday is of a chosen date and hence known the number of days previously arrived at may be increased or decreased by a day or two as may be required to bring it to the correct weekday required.

**JUSTIFICATION** of the addition or subtraction of a day or two is that the thithies due to the anomalistic changes of motion of the Moon need not necessarily begin or end exactly with the beginning or ending of a weekday.



This corrected Aharganam only will have to be used for all further calculations

## TO FIND अहर्गण WITH ENGLISH DATE.

Subtract 1800 from the English year and multiply the balance by 365 242216 day—the length of the tropical year on which basis the English calendar has been worked out and which is responsible for the seasons to appear at regular intervals to mark all civil life. Add to the product the number of days from the 1st of January of the year to the previous day of the date under reference. This divided by 7 will give a remainder which when counted from Thursday as 1 gives the days of the week on the given date.

In cases the day of the week is not known it can be found out as follows. There are twelve constants for the twelve months and they are as follows — January 23 February 3 March 20 April 0 May 24 June 4 July 0 August 8 September 16 October 12 November 20 and December 16

Write the year and add the constant of the month. Increase this by a fourth of itself leaving off any remainder got while dividing by 4. Divide this increased sum by 7 and the remainder will give the weekday beginning from Saturday as 1 corresponding to the 1st date of the month under reference. Having got the weekday of the first of the month the weekday for any other date in that month could be very easily found out.

For dates between 1800 March 1st and 1900 February 28th add 1 weekday to the final result obtained.

Having thus arrived at the correct weekday for the date in question the number of days from 1st January 1800 could now be corrected by adding or subtracting 1 or 2 days as the case may be.

This correction is essential for any fraction involved in the length of the tropical year when multiplied by the number of years may magnify so much as to make a difference of 1 or 2 days more or less which has to be rectified.

### EXAMPLE.

Suppose an individual was born at Mangalore (South Kanara)—Latitude 12°—45' N and Longitude 75° E G M — on 15th July 1912, at 2 A M Indian standard time

## CORRESPONDING TO

Salivahana saka 1834 Paridhavi year (परिधावी) Adhika Ashada month  
(अधिक आषाढमास), New moon (अमावास्या), Sunday (रविवार).

Required to find out the Aharganam and all the planetary positions

—

This is a very complicate example for the birth has taken place in the night after midnight (i.e.) in the early hours of the next morning. According to the European Astronomers and their method of reckoning the above birth is 2 A.M. of 15th July 1912 while the Indian will give it as 14th night and 15th early morning to have perfect synchrony with the weekday which is always supposed to commence with the sunrise that is the birth has taken place on that weekday which had commenced at sunrise on the 14th for our calculation.

### I. INDIAN OR HINDU METHOD.

Saka year birth is	1834
Add	1179
Years after Kaliyuga commencement	5013 ×
multiply by 12	12
	60156
Add number of months from चैत्र to the birth } month आषाढ (take complete months only) }	3
	60159
Additive months = $\frac{1}{12} \times 60159$	= 5013
Adding we get	65172

The birth is in an additive month (i.e.) in an Adhikamasa (अधिकमास), which is followed by the real or the निजमास (Nijamasa) of the same name. As the number of past months will be the same for both of these additive and real months bear the same name the number of months arrived at by the correction applied will really refer to the Nijamasa (निजमास). Hence one month will have to be subtracted if the birth happens to be in Adhikamasa (अधिकमास).

Therefore the number of months	=	62008
	-	1
	=	62007
Multiplying by 30 to convert it to days we have	=	$\times 30$
	=	1860210
No of days past from the last new moon	=	29
		<hr/>
Adding		1860289
subtractive days or क्षयतिथि = $\frac{11}{7} \times 1860289 = 29107$		
Net no of days = $1860289 - 29107 = 1831132$		

This should be the Kaliyuga Aharganam or the no of days since the commencement of Kaliyuga. The balance on dividing by 7 is 2 which when counted from Friday as already stated gives the weekday of birth as Saturday. But we know that the weekday is Sunday. Therefore 1 has to be added to the result already obtained and the correct no of days or अहर्गणं is 1831133.

(It is left to the readers to note that the balance of any, while dividing by 911 or 703 should be ignored even if it may be more than half)

## II ENGLISH DATE METHOD.

Year of birth	1912
subtract always	1800
	<hr/>
No of years from 1800	112
	<hr/>
Length of the tropical year is 365 242216 days	
$\therefore$ No of days for 112 years = $365\ 242216 \times 112 = 40907\ 1282$ .	
Ignoring the fraction we have	40907
No of days from 1st Jan 1912 to the 13th July 1912 is	195
	<hr/>
Adding we get	41102
	<hr/>

This when divided by 7, gives remainder 5 which when counted from Thursday as 1 gives the weekday as Monday but as the day of the week is only a Sunday 1 will have to be subtracted from the above answer 41102. It will be therefore 41101.

Now the no of days from the beginning of the Kaliyuga to 1st January 1800 has already been worked out as 1790032. This अहर्गणं will be the Epoch throughout this book.

Therefore 41101 when added to 1790032 gives 1831133, the result previously obtained by the Indian method

## TEST OF WEEKDAY.

Year of birth	1912
Constant for July the month of birth	0
Adding	<hr/> 1912
½ of 1912, (ignoring remainders if any)	178
Adding	<hr/> 2390 <hr/>

Dividing this by 7, we get remainder 3, which when counted from Saturday as 1, gives Monday as the weekday corresponding to the first date of the month. Therefore 14th was on a Sunday (Vide my pamphlet "A Dive into weekdays"—costing one anna only)

This is an independent method which can be employed with great advantage to fix the weekday of any given date whose corresponding weekday may not be readily available

Therefore to find out the number of days elapsed from our epoch—1st Jan 1800—the method II could be used which will directly give the required result with the necessary weekday correction if any or the method I, but taking care to subtract 1790032 from the final result of the I method

The next stage is to find out the mean heliocentric positions for any given instant. For this, we require the mean heliocentric longitudes at epoch of the planets and their mean angular velocities. The adjoining table gives the elements of all the planets with the periodic time or time of one revolution of each planet about the sun

Subtract the epoch 1790032 if Kaliyuga Aharganam अहर्गण is available or take the no of days from epoch given by II method from the English date corrected to the weekday in question. Both methods will give the same no of days from Epoch. Divide this by the periodic time of each planet. The first quotient will be revolutions—which are not of any use to us. With the balance by using successive multipliers 12 30 60 and 60 respectively and the same periodic time as divisor, get the signs degrees minutes and seconds. These when added to the respective mean position of each of the planets at Epoch give the mean heliocentric position of each

Tables of mean motions have been annexed at the suitable places to save the followers of this book the time and trouble of the labourious divisions

Table showing the elements of the Sun Moon and planets with their mean Longitudes on the 1st January 1800 at mean sunrise at *Tanjore* (Latitude 10 - 47 N and Longitude 79 - 15 E of Greenwich Kaliyuga no of days (कल्याद्यहर्षणं) 1790032

Names	Mean Heliocentric Longitude of planets				Mean Heliocentric Longitudes of Apse of each planet				Mean Heliocentric Longitudes of Node of each planet			
	S gns	Deg	Mts	Secs	S gns	Degs	Mts	Secs	S gns	Degs	Mts	Secs
Sun	8	19	22	58	2	18	25	23	No node for Earth's orbit			
Moon	10	21	40	36	0	24	25	16	0	12	12	49
Mars	7	1	48	54	4	11	18	0	0	26	54	48
Mercury	2	29	19	17	7	23	17	49	0	24	53	36
Jupiter	2	0	50	57	5	20	1	37	2	17	22	44
Venus	4	4	58	30	9	17	42	4	1	23	50	24
Saturn	3	12	2	23	8	8	6	40	3	0	58	16
Uranus	5	2	28	48	10	29	1	41	1	21	43	44
Neptune	6	24	6	20	6	24	15	32	3	18	28	1

Names	Length of semi major axis of orbit	Eccentricity of the orbit	Inclination of orbit to ecliptic	Periodic time
Sun	1 0000	01675	—	965 255374 days
Moon	0025	05490	5° 8'	27 32106 "
Mars	1 5237	09331	1° 51' 1"	686 980 "
Mercury	03871	20561	7° 0'	87 969 "
Jupiter	5 2028	04833	1° 19'	4332 585 "
Venus	7233	00681	3° 23' 6"	224 700 "
Saturn	9 5547	05589	2° 49' 9"	10759 22 "
Uranus	19 2181	04634	0° 46' 4"	30686 84 "
Neptune	30 1096	009	1° 46' 9"	60186 64 "

Names	Annual motion of Apse	Annual motion of Node	Aps of Moon's orbit	Node of Moon's orbit
Sun	+11" 86	Nil	Periodic time of motion of moon's apse is 3239 54051 days (Aps has a forward motion)	Periodic Time of motion of Moon's Nodes is 6793 39474 days (Nodes have a backward motion)
Mars	+16' 86	-22" 74		
Mercury	+6" 14	-6" 82		
Jupiter	+6' 63	-14" 4		
Venus	-1' 52	-19" 14		
Saturn	+16" 10	-16" 57		
Uranus	+3' 22	-32" 28		
Neptune	+1' 19	-10" 68		

6-10°-19-42 Add to this the position of sun at epoch viz 8-19-22-58 we get 2-29-42-40 This is the mean longitude of sun at the birth time

## TABLE OF MEAN MOTION OF SUN.

Periodic time = 365 256371 days

Days	s	°	"	Days	s	°	Days	s	°
1	0	0	59 8	200	6	17 7 19	30000	1	18 16 26
	0	1	58 16	300	9	25 10 59	10000	6	4 21 55
3	0	2	57 25	400	1	4 14 38	50000	10	20 27 24
4	0	3	56 33	500	4	12 48 18	60000	3	6 32 53
5	0	4	55 42	600	7	21 21 55	70000	7	22 38 22
6	0	5	54 50	700	10	29 55 34	80000	0	8 43 50
7	0	6	53 58	800	2	8 29 13	90000	4	24 49 19
8	0	7	53 6	900	5	17 2 53	100000	9	10 54 47
9	0	8	52 14	1000	8	27 36 33	200000	6	21 49 35
10	0	9	51 22	2000	5	21 13 6	300000	4	2 44 22
20	0	19	42 43	3000	2	16 49 39	400000	1	13 39 10
30	0	29	34 5	4000	11	12 26 12	500000	10	24 38 57
40	1	9	25 26	5000	8	8 2 45	600000	8	5 28 45
50	1	19	16 49	6000	5	3 39 18	700000	5	16 23 32
60	1	29	8 13	7000	1	29 15 52	800000	2	27 18 20
70	2	8	59 35	8000	10	24 52 25	900000	0	8 13 7
80	2	18	50 56	9000	7	20 28 57	1000000	9	19 7 55
90	2	28	42 18	10000	4	16 5 29			
100	3	8	33 40	20000	9	2 10 58			

## HINDU METHOD.

Multiply the number of days from epoch by 4 and divide the product by 1461 Leave off the quotient being the number of revolutions With the remainder get signs degrees minutes and seconds Again divide the number of days from epoch by 711 when the quotient obtained will be minutes as before with the remainder get the seconds The difference between the two results will be the mean motion of sun

This requires an empirical correction at the rate of 37-44" for every 100000 days the correction being additive

Let us now try to find out the mean longitude of sun —

# FROM TABLES

	s	d	mts	sec
Position at epoch	8	19	22	58
Motion in 40000 days	6	4	21	55
do 1000 days	8	25	36	33
do 100 days	3	8	33	40
do 1 day	0	0	59	8
do 82153 of a day	0	0	48	28
Adding motion up to the moment of birth	2	29	42	42

This is found to be the same as that previously arrived at, except with a difference of 2 secs of arc, which is apt to occur as the tables have their seconds digits corrected to the nearest integer. The difference is therefore negligible. It is always desirable to take the result of the long division

## BY THE HINDU METHOD.

Dividing the product of the no of days from epoch and 4 by 1461 as indicated in the instructions, we get  $\frac{41101 \times 4}{1461} = 112 \frac{772}{1461}$

Ignoring 112, which is merely the no of revolutions, and converting the fraction,  $\frac{772}{1461}$  to signs etc, we get  $6^{\circ}-10'-13''-33$

Again dividing 41101 by 711 as per instructions we get  $57^{\circ}-48'$ . Subtracting this from the previous result, we get  $6^{\circ}-9'-15''-45$ . Empirical correction for 41101 days is  $\frac{2264^{\circ} \times 41101}{100,000} = 15^{\circ}-30'$ . This is additive as already given in the instructions. Adding therefore we get the mean

motion up to the time of birth from epoch as	s	d	mts	sec
	6	9	31	15
Add to this the mean longitude of sun at epoch, viz	8	19	22	58
We get mean longitude of Sun at the mean sunrise	2	28	54	13
Motion in 82153 of a day	0	0	48	28
Mean longitude of sun at the moment of birth	2	29	42	41

This result is found to tally with those arrived at by the previous two methods

## TO FIND THE POSITION OF THE APSE LINE.

Annual motion of apse line	= (+) $11'' 86$
No of years from 1st Jan 1800 to the 15th July 1912	= 112 54
∴ Motion of Apsē line in 112.54 years	= $11'' 86 \times 112.54 = 1334^{\circ} 6$
	= $1335''$ nearly
	= $22^{\circ} - 15''$
Position at Epoch	= $2^{\circ} - 18' - 25'' - 28''$
∴ Position of Apsē at birth	= $2^{\circ} - 18' - 17'' - 38''$

Subtract the longitude of the sun from that of the apse. Some used to subtract from the planet the perihelion's position instead of the planet from that of the aphelion as we do here. The planet's position minus that of the perihelion is called the *Mean anomaly*. In any case the difference gives the angle between the apse line and that formed by joining the planet (Earth in this case) to the sun. If the position of the planet is subtracted from the aphelion, we get an angle which will be the supplement of that which would be got by subtracting the perihelion from the planet.

In the annexed figure

A — Aphelion

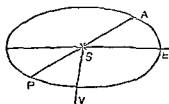
P — Perihelion

S — Sun

E — Earth (or Planet)

Sν — The initial line

towards the direction of the first point of Aries



It will be seen that  $\angle ESA = \angle \nu SA - \angle \nu SE = \text{Aphelion} - \text{Planet}$  and  $\angle PSE = \angle \nu SE + \angle PS\nu = \angle \nu SE + (360' - \angle \nu SP)$  ( $\angle \nu SP$  being counted in the counter-clockwise or positive direction)  $= \angle \nu SE - \angle \nu SP = \text{Planet} - \text{Perihelion}$ . But since ASP is one straight line the angles PSE and ASE will be together equal to 2 rt  $\angle$ s or supplementary.

The former has been adopted by the old Hindu methods while the latter by modern astronomers. There is yet a third school of opinion which requires the subtracting of the Apsē's position from that of the planet in which case the Equation of centre reverses the sign. As the sine fun-

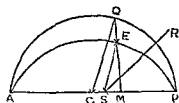


We shall any how adhere to the principle of subtracting the planet's position from that of the Apse of the planet's orbit and call it mean anomaly (मंदकेंद्र).

## TO FIND OUT THE EQUATION OF CENTRE (मंदफलं)

As already stated at the beginning of this chapter according to Kepler's First Law the orbits of the Earth and other planets are all elliptical with the Sun in one of their foci

Let AP be the line of Apes and PSE be the angle described by the Earth after a time  $t$  measured from perihelion where S is the position of the Sun at the focus S of the ellipse



Let SR be the radius supposed to revolve uniformly about S corresponding to mean longitude. Then  $\angle RSP$  is called the *Mean Anomaly* and  $\angle PSE$  the *True Anomaly*.

Now let the  $\perp$  at EM from E to ASP be produced to meet the circle on AP as diameter and C as centre at Q. Then PCQ is called the *Excentric Anomaly*.

The angle ESR is called the equation of centre or मंदफलं, such that True Anomaly = Mean Anomaly + Equation of centre for  $\angle ESP = \angle ESR + \angle RSP$

## RELATION BETWEEN MEAN AND EXCENTRIC ANOMALIES:

Let  $n$  be the mean angular velocity and  $T$  the periodic time whence  $n = \frac{2\pi}{T}$ . Mean anomaly after a time  $t = nt$ . If Q be joined to S then by

Kepler's second law, we have  $\frac{\text{Area of PSQ}}{\text{Area of circle}} = \frac{\text{Area of PSE}}{\text{Area of Ellipse}} = \frac{t}{T} = \frac{nt}{2\pi}$   
but, area of PSQ = sector PCQ -  $\triangle SCQ = \frac{1}{2}a \times a u - \frac{1}{2}a e \times a \sin u$   
where  $u$  is the excentric anomaly,  $a$  the semi-major axis of the ellipse and  $e$  the excentricity of the ellipse

$$\text{We have therefore } \frac{\frac{1}{2}a^2 u - \frac{1}{2}a^2 e \sin u}{\pi a^2} = \frac{nt}{2\pi}$$

$$(1 - e) u - e \sin u = nt \quad \text{I}$$

[NOTE Area of a sector =  $\frac{1}{2}$  the radius  $\times$  the bounding arc  
 $= \frac{1}{2} a \times a \times$  the no of radians contained in the angle  
 and  $\triangle SCQ = \frac{1}{2} CS \times \perp ar$  from Q on CS  
 $= \frac{1}{2} CS \times CQ \sin \angle QCS$   
 $= \frac{1}{2} a e \times a \sin u$  ]

The fact that  $CS = a e$  belongs to the realms of Analytical Conics on the properties of the Ellipse a full discussion of which we do not propose to enter into

## RELATION BETWEEN TRUE & EXCENTRIC ANOMALY.

If  $\theta$  (theeta) be the true anomaly  $SE \cos \theta = CM - CS$

$\frac{a(1-e^2)}{1+e \cos \theta} \cos \theta = a \cos \angle QCM - a e = a \cos u - a e$  (for  $SE = \frac{l}{1+e \cos \theta}$   
 by the polar equation of conics and  $l = a(1-e^2)$  for an ellipse)

$$a(1-e^2) \cos \theta = a(\cos u - e)(1+e \cos \theta)$$

$$(1-e)(1-e) \cos \theta = \cos u - e + e \cos \theta \cos u - e^2 \cos \theta$$

$$\therefore (1-e \cos u) \cos \theta = \cos u - e$$

$$(1-e) \cos \theta = \frac{\cos u - e}{1 - e \cos u}$$

$$\frac{1 - \cos \theta}{1 + \cos \theta} = \frac{(1+e)(1 - \cos u)}{(1-e)(1 + \cos u)}$$

$$(1-e) \tan \frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{u}{2} \quad \text{II}$$

Thus the excentric anomaly serves as a link between mean and true anomalies. We always require  $\theta$  in terms of  $t$ . This is done by employing a succession of trials to approximate to the value of  $u$  in I and then to determine  $\theta$  by II

A knowledge of Differential Calculus on the expansion of  $u$  in ascending powers of  $e$  is essential but we have neither the time nor the place to dwell on it here but merely have to use the expansion already arrived at by such successive approximations

The result to the third power of  $e$  is  $\theta = nt + 2e \sin nt + \frac{5e^2}{4} \sin 2nt$

$$+ \frac{e^3}{12} (18 \sin 3nt - \dots)$$

. Equation of centre or मरकल =  $2e \sin nt + \frac{5e}{4} \sin 2nt +$  negligible terms  $e$  being very small for all practical purposes the cube and higher powers of  $e$  may be neglected

This formula applies to all planets and can be used for determining the equation of centre corresponding to any mean position of the planet provided we know the mean anomaly and also the excentricity of the planetary orbit

A brief note on the nature of Hindu Astronomy on equation of centre will not be out of place First of all the Hindus have taken the equation only as far as the first term and secondly the value of the excentricity also is slightly higher than its present value with the result that the maximum equation of centre becomes  $128-57$  while its real value as found out by modern astronomers is in the neighbourhood of  $115-19$

By taking only one term of the equation of centre they use the same table for different sets of mean anomaly by taking  $nt$  ( $180-nt$ ) ( $nt$   $180$ ) or ( $360-nt$ ) as the argument according as the mean anomaly is in the I II III or IV quadrants care being taken with due regard to the sign of the equation of centre with respect to the mean anomaly

Thus the set of Hindu tables on the equation of centre of Earth and other planets are not giving true values and it is high time they are rectified to be of use and ready acceptance We propose to rectify it thus —

Equation of centre as derived by formula previously is  $2e \sin nt + \frac{5e^2}{4} \sin 2nt$  where  $nt$  is the mean anomaly with reference to the Perihelion and equal to Planet minus Perihelion But the mean anomaly which we have already defined is Aphelion minus Planet and hence the mean anomaly we use will be only the supplement of that in the formulae That is if  $nt$  be our mean anomaly ( $180-nt$ ) will have to be taken as the mean anomaly for purposes of the formula of equation of centre

$$\begin{aligned} \text{Equation of centre} &= 2e \sin nt - \frac{5e^2}{4} \sin 2nt \\ &= \frac{e \sin nt}{2} (4 - 5e \cos nt) \end{aligned}$$

In the case of Sun (Earth)  $e = 0.1675$

$$\text{Equation of centre} = \frac{0.1675}{2} \sin nt \times (4 - 0.8375 \cos nt)$$

This equation of centre will be only in radians for the mean velocity is angular. Therefore circular measure when multiplied by 206265 the number of seconds in a radian will reduce to seconds of arc of the Sexagesimal measure

Multiplying the equation of centre in the last para by 206265 we get  
Equation of centre in seconds of arc

$$= 0.08375 \times 206265 (4 - 0.8375 \cos nt) \sin nt$$

$= 1727.5 (4 - 0.8375 \cos nt) \sin nt$  But according to the Hindu tables the equation of centre will be  $7737 \sin nt$

The ratio which will therefore reduce the value of the equation as per the Hindu table to that of the Modern table is  $\frac{17 (1 - 0.2094 \cos nt)}{19}$  very nearly

Having thus found the true anomaly if the longitude of Apse be added we get the true heliocentric longitude for the anomaly is only the angle between the radius vector of the planet and the line of Apsides

In practice the mean anomaly is found out by subtracting the mean heliocentric longitude from the longitude of Apse. The equation of centre is determined from that mean anomaly and applied to the mean longitude of the planet positive or negative according as the mean anomaly is less than or greater than  $180^\circ$

Let us now revert to the example taken —

		s	deg	mts	secs
Mean longitude of Sun	=	2	29	42	40
do of Apse	=	2	18	47	38

$$\begin{aligned} \text{Mean anomaly} &= \text{Apse} - \text{Sun} \\ &= 11^\circ - 19' - 4'' - 58''' = 349^\circ - 4'' - 58''' \end{aligned}$$

## TRIGONOMETRICAL METHOD.

Equation of centre in seconds  $= 1727.5 (4 - 0.8375 \cos nt) \sin nt$   
where  $nt = 349^\circ - 4'' - 58'''$  in this case

$$\sin nt = \sin 349^\circ - 4'' - 58'''$$

$$= \sin (360^\circ - 10^\circ - 55' - 2'')$$

$$= -\sin 10^\circ - 55' - 2'' \text{ the given } \angle \text{ being in the IV quadrant}$$

To find now  $\sin 10^\circ-55'-2''$  enter the table of Trigonometrical ratios at the end of this book and find  $\sin 10^\circ = 1736482$  and 171608 as the difference for  $1''$

$$\therefore \text{Proportional difference for } 55'-2'' = \frac{55'-2''}{60'} \times 171608 = 157402$$

$$\begin{aligned} \therefore \sin 10^\circ-55'-2'' &= 1736482 + \\ &\quad 157402 \\ &= \underline{\underline{1893884}} \end{aligned}$$

$$\therefore \sin 349^\circ-4'-58'' = -1893884$$

$$\begin{aligned} \text{So also, } \cos 349^\circ-4'-58'' &= \cos (360^\circ-10^\circ-55'-2'') \\ &= \cos 10^\circ-55'-2'' \text{ being IV quadrant} \\ &= \sin (90^\circ-10^\circ-55'-2'') \\ &= \sin (79^\circ-4'-58'') \end{aligned}$$

$$\sin 79^\circ = 9816272, \text{ difference for } 1'' = 31806$$

$$\therefore \text{difference for } 4'-58'' = \frac{31806 \times 4'-58''}{60} = 2633$$

$$\begin{aligned} \therefore \sin 79^\circ-4'-58'' &= 9816272 + \\ &\quad 2633 \\ &= \underline{\underline{9818905}} \end{aligned}$$

$$(i.e.) \cos 349^\circ-4'-58'' = 9818905$$

Substituting these values for  $\sin nt$  and  $\cos nt$ , we get the Equation of centre as being  $= -1893884 \times 1727.5 \times (4 - 0.8375 \times 9818905)$

$$= -1727.5 \times 1893884 (4 - 0.82232)$$

$$= -1727.5 \times 1893884 \times 3.917768$$

$$= -128176 \text{ secs}$$

$$= -1282 \text{ secs} = \underline{\underline{-21'-22''}}$$

$$\text{Mean longitude} = 2^\circ-29'-42''-40$$

$$\text{Equation of centre} = \underline{\underline{\text{minus } 21'-22''}}$$

$$\therefore \text{True longitude} = \underline{\underline{2^\circ-29'-21'-18''}}$$

## TO FIND THE RADIUS VECTOR OF THE EARTH.

The radius vector,  $r$  is usually found by the Polar equation of a conic according to Analytical comets viz  $r = \frac{l}{1+e \cos \theta}$ , where  $\theta$  is the true

anomaly as per the usual convention. But consistent with the definition of mean true anomaly we have laid in here the formula will be  $r = \frac{l}{1-e \cos \theta} = \frac{a(1-e^2)}{1-e \cos \theta}$  where  $e$  is the excentricity and  $a$  the semi-major axis of the earth's orbit being equal to 1

$$\begin{aligned} \text{Radius vector} &= \frac{1-e^2}{1-e \cos \theta} = \frac{(1+e)(1-e)}{1-e \cos \theta} \\ &= \frac{1.01675 \times 98325}{1-0.01675 \cos \theta} = \frac{99972}{1-0.01675 \cos \theta} \end{aligned}$$

$\theta$  is the True anomaly as already stated. In the present example  
 True anomaly = Apse — True longitude  
 $= (2^s-18^m-47^s-38'') - (2^s-29^m-21^s-18'')$   
 $= 11^s-19^m-26^s-20''$   
 $= 349^{\circ}-26'-20''$

$$\therefore \text{Radius vector} = \frac{99972}{1-0.01675 \cos (349^{\circ}-26'-20'')}$$

$$\begin{aligned} \cos 349^{\circ}-26'-20'' &= \cos (360-10^{\circ}-33'-40'') \\ &= \cos 10^{\circ}-33'-40'' \end{aligned}$$

$$\cos 10^{\circ} = 9848078, \text{ difference for } 1'' = -91806$$

$$\therefore \text{ difference for } 33'-40'' \text{ is } \frac{33'-40' \times -91806}{60} = -17674$$

$$\begin{aligned} \therefore \cos 10^{\circ}-33'-40'' &= \frac{9848078-17674}{\phantom{000000}} \\ &= \underline{\underline{9830404}} \end{aligned}$$

$$\therefore \cos 349^{\circ}-26'-20'' = 9830404$$

$$\begin{aligned} \therefore \text{ radius vector} &= \frac{99972}{1-0.01675 \times 9830404} \\ &= \frac{99972}{1-0.016465} = \frac{99972}{983535} \\ &= \underline{\underline{1.01616}} \end{aligned}$$

As it is thus very difficult to calculate the equation of centre and the radius vector, the annexed table could be used. The radius vector could be found out directly from the mean anomaly. The Heliocentric velocity at any instant is also given which has been calculated thus

Velocity in seconds is usually got by differentiating the expression for  $\theta$  with respect to the variable  $t$  whence velocity

$$\begin{aligned}
 &= n + 2 n e \cos nt + \frac{5e^2 n}{2} \cos 2 nt \\
 &= n \left\{ 1 + 2e \cos nt + \frac{5e^2}{2} (2 \cos^2 nt - 1) \right\} \\
 &= n \left\{ \left( 1 - \frac{5e^2}{2} \right) + 2e \cos nt + 5e^2 \cos^2 nt \right\} \\
 &= 3545.4 + \cos nt (119 + 5 \cos nt) \text{ for in the case of Sun}
 \end{aligned}$$

$n = 59.8$  but our mean anomaly is the angle equal to (Apse — Planet) and hence the velocity will be  $3545 - \cos nt \times (119 - 5 \cos nt)$

In the present example

$$\text{velocity} = 3545 - \cos 349^\circ - 4' - 58'' (119 - 5 \cos 349^\circ - 4' - 58'')$$

$$\cos 349^\circ - 4' - 58'' = \cos (360^\circ - 10^\circ - 55' - 2'') = \cos 10^\circ - 55' - 2''$$

$$\cos 10^\circ = 9848078 \quad \text{difference for } 1'' = -31806$$

$$\text{difference for } 55' - 2'' = -29173$$

$$\cos 10^\circ - 55' - 2'' = \underline{\underline{9818905}}$$

$$\text{Velocity} = 3545 - 9818905 (119 - 5 \times 9818905)$$

$$= 3645 - 9818905 (119 - 49094525)$$

$$= 3545 - 9818905 \times 1140905475$$

$$= 3545 - 112 = \underline{\underline{3433.57 - 13}}$$

## FROM TABLES

To find out the Equation of Centre Heliocentric Velocity and Radius vector of the (Earth) Sun the argument will be the mean anomaly as defined by us viz (Apse — Planet) When the mean anomaly thus defined is less than  $180^\circ$  the tables may be entered into directly but when greater than  $180^\circ$  the same must be subtracted from  $360^\circ$  when the balance will be the argument to be used In the latter case it should be *unfailingly* noted down that the sign of the equation of centre will be negative while the Heliocentric Velocity and Radius vector always remain positive

Table showing Equation of Centre, Heliocentric Velocity and Radius vector of Sun's (Earth's) motion.

Each Degree of Argument	Equation of Centre		Heliocentric Velocity		Radius Vector	Each Degree of Argument	Equation of Centre		Heliocentric Velocity		Radius Vector
0	0	0	57	11	1 0168	42	75	57	57	41	1 0126
1	1	58	57	11	1 0168	43	77	26	57	42	1 0124
2	3	56	57	11	1 9168	44	78	53	57	43	1 0122
3	5	54	57	12	1 0168	45	80	18	57	44	1 0120
4	7	52	57	12	1 0168	46	81	42	57	45	1 0118
5	9	50	57	12	1 0167	47	83	5	57	47	1 0116
6	11	47	57	12	1 0167	48	84	27	57	48	1 0114
7	13	45	57	12	1 0167	49	85	46	57	50	1 0112
8	15	43	57	13	1 0166	50	87	5	57	51	1 0109
9	17	40	57	13	1 0166	51	88	23	57	53	1 0107
10	19	37	57	13	1 0165	52	89	38	57	54	1 0105
11	21	33	57	13	1 0165	53	90	52	57	56	1 0102
12	23	28	57	14	1 0164	54	92	5	57	57	1 0100
13	25	23	57	14	1 0163	55	93	16	57	59	1 0098
14	27	18	57	15	1 0163	56	94	26	58	0	1 0095
15	29	12	57	15	1 0162	57	95	34	58	2	1 0093
16	31	6	57	16	1 0161	58	96	40	58	4	1 0091
17	33	0	57	16	1 0161	59	97	45	58	5	1 0088
18	34	53	57	17	1 0160	60	98	49	58	7	1 0086
19	36	46	57	17	1 0159	61	99	47	58	9	1 0083
20	38	38	57	18	1 0158	62	100	43	58	11	1 0081
21	40	30	57	18	1 0157	63	101	38	58	12	1 0078
22	42	20	57	19	1 0156	64	102	32	58	14	1 0076
23	44	10	57	20	1 0155	65	103	24	58	16	1 0073
24	45	59	57	20	1 0154	66	104	16	58	18	1 0071
25	47	47	57	21	1 0153	67	105	6	58	19	1 0068
26	49	35	57	22	1 0151	68	105	55	58	21	1 0066
27	51	21	57	23	1 0150	69	106	48	58	23	1 0063
28	53	6	57	24	1 0149	70	107	30	58	25	1 0060
29	54	51	57	25	1 0147	71	108	12	58	27	1 0057
30	56	35	57	26	1 0146	72	108	51	58	29	1 0054
31	58	17	57	27	1 0145	73	109	29	58	31	1 0051
32	59	57	57	28	1 0143	74	110	5	58	33	1 0049
33	61	37	57	29	1 0142	75	110	39	58	35	1 0046
34	63	17	57	30	1 0140	76	111	12	58	37	1 0043
35	64	55	57	31	1 0139	77	111	43	58	39	1 0041
36	66	33	57	32	1 0137	78	112	13	58	40	1 0038
37	68	10	57	33	1 0135	79	112	41	58	42	1 0035
38	69	46	57	35	1 0134	80	113	7	58	44	1 0032
39	71	21	57	36	1 0132	81	113	31	58	46	1 0029
40	72	56	57	38	1 0130	82	113	51	58	48	1 0026
41	74	28	57	39	1 0128	83	114	9	58	50	1 0023



Each Degree of Argument	Equation of Centre			Heliocentric Velocity		Radius Vector	Each Degree of Argument	Equation of Centre			Heliocentric Velocity		Radius Vector
84	114	26	58	51	1.0020	129	90	44	60	21	9896		
85	114	40	58	53	1.0017	130	89	29	60	23	9894		
86	114	52	58	56	1.0015	131	88	11	60	25	9891		
87	115	1	58	58	1.0012	132	86	50	60	26	9889		
88	115	9	59	00	1.0009	133	85	29	60	28	9887		
89	115	14	59	3	1.0006	134	84	6	60	30	9885		
90	115	16	59	5	1.0003	135	82	42	60	32	9883		
91	115	18	59	7	1.0000	136	81	16	60	33	9881		
92	115	19	59	9	.9997	137	79	49	60	35	9879		
93	115	16	59	12	.9994	138	78	21	60	37	9877		
94	115	12	59	14	.9991	139	77	51	60	38	9875		
95	115	5	59	16	.9988	140	75	20	60	40	9873		
96	114	55	59	18	.9985	141	73	48	60	41	9871		
97	114	44	59	20	.9982	142	72	18	60	43	9869		
98	114	31	59	23	.9979	143	70	37	60	44	9867		
99	114	15	59	25	.9976	144	69	0	60	45	9865		
100	113	57	59	27	.9973	145	67	21	60	46	9863		
101	113	38	59	29	.9971	146	65	41	60	47	9862		
102	113	15	59	31	.9968	147	63	59	60	49	9860		
103	112	49	59	33	.9965	148	62	15	60	50	9858		
104	112	24	59	35	.9962	149	60	30	60	51	9857		
105	111	56	59	37	.9959	150	58	44	60	52	9855		
106	111	26	59	38	.9956	151	56	56	60	53	9853		
107	110	54	59	40	.9954	152	55	7	60	54	9852		
108	110	20	59	42	.9951	153	53	17	60	55	9851		
109	109	44	59	44	.9948	154	51	26	60	56	9849		
110	109	5	59	46	.9945	155	49	35	60	57	9848		
111	108	26	59	48	.9942	156	47	43	60	58	9847		
112	107	43	59	50	.9940	157	45	51	60	59	9846		
113	106	59	59	52	.9937	158	43	58	61	0	9845		
114	106	13	59	54	.9934	159	42	4	61	1	9844		
115	105	25	59	56	.9931	160	40	9	61	2	9843		
116	104	35	59	58	.9929	161	38	14	61	2	9842		
117	103	43	60	00	.9926	162	36	18	61	3	9841		
118	102	49	60	2	.9923	163	34	21	61	3	9840		
119	101	53	60	4	.9921	164	32	24	61	4	9839		
120	100	55	60	6	.9918	165	30	26	61	4	9838		
121	99	53	60	8	.9915	166	28	27	61	5	9837		
122	98	48	60	9	.9913	167	26	28	61	5	9837		
123	97	43	60	11	.9911	168	24	28	61	6	9836		
124	96	36	60	13	.9908	169	22	27	61	6	9835		
125	95	28	60	15	.9905	170	20	26	61	7	9835		
126	94	19	60	16	.9903	171	18	25	61	7	9834		
127	93	9	60	18	.9900	172	16	24	61	7	9834		
128	91	57	60	20	.9898	173	14	22	61	7	9833		

Each Degree of Argument	Equation of Centre		Heliocentric Velocity		Radius Vector	Each Degree of Argument	Equation of Centre		Heliocentric Velocity		Radius Vector
174	12	19	61	7	9838	178	4	8	61	8	9832
175	10	17	61	8	9833	179	2	4	61	8	9832
176	8	14	61	8	9832	180	0	0	61	9	9832
177	6	11	61	8	9832						

## EQUATION OF CENTRE FROM THE TABLES.

$$\begin{array}{rcl}
 \text{Mean Anomaly (Apse sun)} & & s \quad d \quad m \quad s \quad e \quad c \quad s \\
 & = & 11-19-4 \quad 58 \\
 \left. \begin{array}{l} \text{This is more than } 180^\circ \\ \text{Subtracting this from } 360^\circ \text{ as per} \\ \text{instructions we get} \end{array} \right\} & & \text{or } 349-4 \quad 58 \\
 & & 360-0-0 \text{ minus} \\
 & & 349-4-58 \\
 & & \hline
 \text{Argument for entering into the table} & = & 10-55-2
 \end{array}$$

$$\begin{array}{rcl}
 \text{From tables equation of centre for } 10^\circ \text{ is} & 19-37 \\
 \text{do} & \text{for } 11^\circ \text{ is} & 21-33
 \end{array}$$

Difference for 1 is 1-56" (+)

$$\text{Difference for } 55-2 \text{ is } \frac{116}{60} \times 55-2 = 1-46$$

$$\begin{array}{rcl}
 \text{Equation of centre for } 10^\circ-55-2 \text{ is} & 19-37'' \\
 & + & 1-46'' \\
 & \hline
 & 21-23
 \end{array}$$

But since the mean anomaly is more than 6 signs or 180 this equation of centre is negative

The equation of centre is 21-23 (-) Subtracting this equation since negative from the mean longitude we get

$$\text{True Longitude} = \left\{ \begin{array}{l} 2-29^\circ-42-40'' \\ \text{minus } 21-23 \\ \hline 2^\circ-29^\circ-21'-17'' \end{array} \right.$$

Against the same argument the velocity and Radius vector are seen from the Table to be 57-13 and 10165 respectively These have been already arrived at by purely Trigonometrical methods independent of the tables

# HINDU TABLES OF SUN'S EQUATION OF CENTRE.

॥ गविन्दज्यापदकानि ॥

Degree	Equation of Centre		Equation of Velocity		Degree	Equation of Centre		Equation of Velocity		Degree	Equation of Centre		Equation of Velocity	
0	0	0	2	13	32	68	19	1	52	64	115	53	0	57
1	2	15	2	13	33	70	13	1	51	65	116	51	0	55
2	4	30	2	13	34	72	5	1	50	66	117	47	0	53
3	6	45	2	13	35	73	57	1	48	67	118	41	0	51
4	9	0	2	13	36	75	46	1	46	68	119	32	0	49
5	11	14	2	13	37	77	35	1	45	69	120	22	0	47
6	13	28	2	13	38	79	22	1	44	70	121	9	0	45
7	15	43	2	12	39	81	8	1	43	71	121	55	0	42
8	17	56	2	11	40	82	52	1	41	72	122	38	0	40
9	20	10	2	11	41	84	35	1	40	73	123	19	0	38
10	22	23	2	10	42	86	16	1	38	74	123	57	0	36
11	24	36	2	10	43	87	55	1	36	75	124	33	0	34
12	26	48	2	9	44	89	33	1	35	76	125	7	0	32
13	29	0	2	9	45	91	9	1	33	77	125	39	0	29
14	31	11	2	8	46	92	44	1	32	78	126	8	0	26
15	33	22	2	8	47	94	17	1	30	79	126	35	0	24
16	35	32	2	7	48	95	48	1	29	80	126	59	0	22
17	37	41	2	6	49	97	18	1	28	81	127	22	0	20
18	39	50	2	4	50	98	45	1	25	82	127	42	0	17
19	41	58	2	4	51	100	11	1	23	83	127	59	0	15
20	44	5	2	4	52	101	35	1	21	84	128	15	0	12
21	46	12	2	4	53	102	57	1	19	85	128	27	0	9
22	48	18	2	4	54	104	18	1	17	86	128	38	0	6
23	50	23	2	2	55	105	37	1	15	87	128	46	0	4
24	52	26	2	1	56	106	54	1	13	88	128	52	0	2
25	54	29	1	59	57	108	8	1	11	89	128	56	0	1
26	56	31	1	58	58	109	21	1	9	90	128	57	—	—
27	58	32	1	58	59	110	31	1	8					
28	60	31	1	57	60	111	40	1	6					
29	62	30	1	56	61	112	46	1	4					
30	64	27	1	55	62	113	51	1	1					
31	66	24	1	53	63	114	53	0	59					

## INSTRUCTIONS TO USE THE HINDU TABLE.

The Hindu table of Sun's equation of centre gives the equation of centre and equation of Velocity up to 90° of the argument. For, as already told, the Hindus have taken only one term of the formula of Equation of

centre which is a sine function gradually increasing in the I quadrant decreasing in the II quadrant and again reversing the same during the III and IV quadrants Therefore a single table of  $90^\circ$  of argument will suffice all their requirements

For  $0^\circ$  to  $90^\circ$  of anomaly use the tables as given

$90^\circ$  to  $180^\circ$  of anomaly subtract the anomaly from  $180^\circ$  the maximum for II quadrant and the difference will be the argument to refer the table

$180^\circ$  to  $270^\circ$  of anomaly subtract  $180^\circ$  from the anomaly and the balance will be the argument

$270^\circ$  to  $360^\circ$  of anomaly subtract the anomaly from  $360^\circ$  the balance will be the argument required

After the equation of centre is got from the table to bring it to the modern value the multiplier already mentioned in a previous para viz  $\frac{1}{10} (1 - 0.2094 \cos nt)$  should be applied to it The rectified equation of centre should be added to or subtracted from the mean longitude according as the Mean anomaly is less than  $180^\circ$  (मेपादि) or greater than  $180^\circ$  (तुलादि).

*As for Velocity* The equation of Velocity after being obtained from the table should be similarly applied to by the multiplier

$$\frac{3 + \cos nt \times (119 - 5 \cos nt)}{1.13 \cos nt}$$

This rectified equation of velocity should be added to or subtracted from the mean velocity  $59 - 8''$  according as the mean anomaly lies between  $90^\circ$  to  $270^\circ$  (कक्यादि) or between  $270^\circ$  to  $90^\circ$  (मकरादि)

Expressing briefly in Hindu method for Equation of Centre mean anomaly has to be considered as मेपादिघनं or तुलादि ऋण and for Equation of Velocity कक्यादिघन or मकरादि ऋण

## EXAMPLE.

Mean longitude of Sun previously arrived at by Hindu method	} is	s	deg	mts	secs
Longitude of Apse		2	29	42	40
(Apse—Sun) or mean anomaly		2	18	47	38
		11	19	4	58

This is in the IV quadrant, being greater than  $270^\circ$  but less than  $360^\circ$ . Therefore, the argument for referring to the table is got by subtracting the mean anomaly from  $360^\circ$ . It will be  $360^\circ - 349^\circ - 4' - 58''$  or  $10^\circ - 55' - 2''$ . With this argument now referring to the tables we find as follows

For $10^\circ$	Eqn of centre	$22' - 23''$	Eqn of Velocity	$2' - 10''$
For $11^\circ$	do	$24' - 36''$	do	$2' - 10''$
Difference for $1^\circ$	do	$2' - 13''$		Nil
$\therefore$ Proportional diff for $55' - 2''$ }	is	$2' - 3''$		Nil
$\therefore$ Required eqn, of centre = $\left\{ \begin{array}{l} 22' - 23'' \\ 2' - 3'' \end{array} \right. +$				
$= \underline{\underline{24' - 26''}}$				

$$\text{Equation of Velocity} = 2 - 10''$$

The equation of centre will be negative as the mean anomaly is more than  $180^\circ$  or सुलादि, and the Equation of Velocity is also negative as the mean anomaly is between  $270^\circ$  and  $90^\circ$  or मकरादि.

$$\begin{aligned} \text{Multiplier for Equation of Centre is } \frac{17}{15} (1 - 0.2094 \cos nt) \\ = \frac{17}{15} (1 - 0.2094 \times 9819) \\ = \frac{17}{15} (1 - 0.20559) = \frac{17}{15} \times 0.79441 \\ = 0.876342 \end{aligned}$$

$$\begin{aligned} \therefore \text{Correct equation of centre} &= -(24' - 26'') \times 0.876342 \\ &= -(21' - 24'') \end{aligned}$$

$$\begin{aligned} \therefore \text{True longitude} &= \left\{ \begin{array}{l} 2^\circ - 29^\circ - 42' - 40'' \\ \text{minus } 21' - 24'' \end{array} \right. \\ &= \underline{\underline{2^\circ - 29^\circ - 21' - 16''}} \end{aligned}$$

$$\begin{aligned} \text{Multiplier for equation of Velocity is } \frac{3 + \cos nt (119 - 5 \cos nt)}{132 \cos nt} \\ = \frac{3 + 9819(119 - 5 \times 9819)}{132 \times 9819} \\ = \frac{3 + 9819 \times 114.1905}{132 \times 9819} = \frac{115.11}{130.60} \end{aligned}$$

the First point of Aries is  $126^{\circ}$ . Thus the tropical longitude has increased by  $148^{\circ}-11'$  minus  $126^{\circ}$ , which is equal to  $22^{\circ}-11'$ . The number of years required for this increase is 1580.82 years at the lower rate and 1575.86 years at the mean rate, the higher rate being far higher than the present rate, having been ignored.

Therefore, the year when the moving First point of Aries coincided with Virtual fixed Aries or in other words the year when the precession was zero is 1881 minus 1575.86 or 305.14 A.D. Motion of Ayanamsa or Amount of precession from 305.14 A.D. to 1800 A.D. at the rate of 50.6773 per year is

$$\begin{aligned} &= 50.6773 \times (1800 - 305.14) \text{ secs} \\ &= 50.6773 \times 1494.86 \text{ secs} \\ &= 75755 \text{ secs} \\ &= 21^{\circ}-2'-35'' \end{aligned}$$


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This is the amount of precession on the 1st January 1800

## TO FIND OUT THE PRECESSION AT ANY SUBSEQUENT DATE :—

The present all-accepted value of precession per year is 50.2286 though the Suryasiddhanta has taken 54 and the Graha Laghava 60 as its value. For our calculation we will take only the rate 50.2286 for years after our epoch.

Therefore if the precession at any subsequent date is required multiply the rate 50.2286 by the number of years from epoch. The result will be the amount of precession for the interval after the epoch and this added to the amount already preceded up to the epoch viz  $21^{\circ}-2'-35''$ , will give the amount of precession required.

### EXAMPLE.

No. of years after epoch	= 112.54
Rate of precession per year	= $50''$ 2286
∴ Amount of precession	= $112.54 \times 50''$ 2286
	= 5653 sec
	= $1^{\circ}-34'-13''$
Amount of Precession at Epoch	= $21^{\circ}-2'-35''$
	<hr/>
∴ Amount of precession on 15th } July 1912	= $22^{\circ}-36'-48''$
	<hr/>

# Chapter VIII

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## TIME—THE VARIOUS KINDS—EQUATION OF TIME.

For all astronomical purposes the sidereal day is one of the principal units of time. It begins at the instant when the First Point of Aries is on the meridian and at any subsequent instant the sidereal time will be the hourangle of the First Point of Aries measured westwards.

A *solar day* is the interval between two successive transits of the centre of the Sun on the meridian. The sun changes his R. A. as he is advancing eastwards among the stars at an angle of about 1° per day and therefore the earth will have to turn one more time about its axis to complete a solar day, which will consequently be about 4 minutes longer than a sidereal day.

Thus the solar time at any instant is the hourangle of the sun's centre reckoned westward from 0 hrs to 24 hrs. This is called the *Apparent Solar Time* and it is only this time that is indicated by a sundial.

If the sun's motion in right ascension were uniform the solar days will all be equal to one another, but this is not the case. In the first place the sun's motion in its own orbit is not uniform, due to the eccentricity of the orbit and secondly even if it were, the corresponding motion in R. A. would not be uniform due to the inclination of the orbit to the equator on which R. A.'s are measured.

The solar day marking the recurrence of light and darkness is obviously that on which man in civil life must regulate his time although the want of uniformity mentioned above hinders us from employing it as a measuring unit. We may however obtain an uniform measure of time depending upon the sun in the following manner.

Conceive an imaginary body called the Dynamical Mean Sun to move along the equator with the mean angular velocity of the true sun. The days marked by this mean sun will be uniform and equal and exactly the average of all the solar days during the year. Therefore a clock whose motion is necessarily uniform may be regulated on the mean sun. To have connection between the two suns, we must establish the starting point of the mean sun and it will be convenient so to choose that the mean solar time and the apparent solar time may never be widely separated.

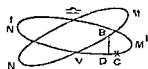
Conceive another imaginary body say a star to have the same uniform angular velocity as the mean sun but to move along the ecliptic instead of the equator and to pass through the perigee at the same time as the true sun. Then the motion of the mean sun is so adjusted that it may pass through the first point of Aries at the same time as the star.

It could be seen that the connection between the two suns may be expressed by saying that the R.A. of the mean sun is equal to the mean longitude of the true sun because the mean longitude of the true sun is the longitude of the supposed star.

The difference between the Apparent time and Mean time at any instant is called the *Equation of Time*. It is considered as the correction to be applied to the former to obtain the latter and is therefore called positive when mean noon precedes true noon and vice versa.

It is obvious that the equation of Time is the value expressed in Time the angle between the declination circles of the true and mean suns. To have clear idea of the variations in the equation of time we may consider the individual causes that give rise to the equation and the algebraical sum of the two effects will give us the cumulative effect.

## I. CONSIDER THE INCLINATION OF THE ECLIPTIC TO THE EQUATOR.



Let us suppose the sun to describe the orbit  $T^{\circ}M^{\circ}N^{\circ}$  with uniform angular velocity and the mean sun describing the equator  $T^{\circ}M^{\circ}N^{\circ}$  with the same velocity the two pass

ing thro  $T^{\circ}$  at the same time

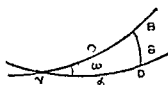
When the true sun is at B the mean sun will be at C where  $T^{\circ}B = T^{\circ}C$  and if BD is the declination circle through B CD will measure the equation of time due to this cause. Further C and D will coincide only at the equinoxes and the solistices.

From  $T^{\circ}$  to M C will be in advance of D and from M to  $\simeq$  C will be behind D. Thus from  $T^{\circ}$  to M the apparent noon will precede mean noon and the equation of time will be subtractive. So also it is additive from solistices to equinoxes (i.e.) from M to  $\simeq$ .



This effect on equation of time due to the inclination of the orbit to the equator is called the "ग्रहभ्रम" in Hindu Astronomy. It is found out by solving the  $rt \angle d$  spherical triangle

Let  $\Upsilon B$  be the longitude of the true Sun and equal to  $\odot$ ,  $\Upsilon D$  be the R.A. of the true Sun and equal to  $\alpha$  and  $\angle B\Upsilon D = \omega$  the obliquity of the ecliptic. Since  $BD$  is  $\perp$ ar to the equator being a declination circle  $\triangle B\Upsilon D$  is  $rt \angle d$ .



Sine of middle part

= product of tangents of adjacents (by Napier's rules for spherical  $rt \angle d$  vide chapter II)

If  $\angle B\Upsilon D$  is taken as the middle part  $\Upsilon D$  and  $\Upsilon B$  will be the adjacents and its Napier's parts are  $(90^\circ - \hat{B}\Upsilon D)$   $\Upsilon D$  and  $(90^\circ - \Upsilon B)$  (see chapter II)

$$\therefore \sin (90^\circ - \hat{B}\Upsilon D) = \tan \Upsilon D \times \tan (90^\circ - \Upsilon B)$$

$$\begin{aligned} \text{i.e. } \cos \omega &= \tan \Upsilon D \times \cot \Upsilon B \\ &= \tan \alpha \times \cot \odot \end{aligned}$$

$$\text{for } \sin (90^\circ - \omega) = \cos \omega$$

$$\text{and } \tan (90^\circ - \Upsilon B) = \cot \Upsilon B = \cot \odot$$

$$\therefore \tan \alpha = \frac{\cos \omega}{\cot \odot} = \cos \omega \tan \odot$$

$$\begin{aligned} \therefore \tan (\odot - \alpha) &= \frac{\tan \odot - \tan \alpha}{1 + \tan \odot \tan \alpha} \\ &= \frac{2 \tan \odot \sin^2 \frac{\omega}{2}}{1 + \tan^2 \odot \cos \omega} \end{aligned}$$

$$\therefore (\odot - \alpha) = \tan^{-1} \left[ \frac{2 \tan \odot \sin^2 \frac{\omega}{2}}{1 + \tan^2 \odot \cos \omega} \right]$$

A table of equation of time due to the obliquity of the ecliptic is here-with appended. The equation is negative for  $0^\circ$  to  $90^\circ$  and  $180^\circ$  to  $270^\circ$  of the tropical longitude and positive for  $90^\circ$  to  $180^\circ$  and  $270^\circ$  to  $360^\circ$  of the same

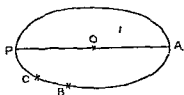
Briefly, it is negative in the odd and positive in the even quadrants of the tropical longitude Expressing in sanskrit it will be "ओजपदेरुणे and समपदे धने "

TABLE OF EQUATION OF TIME DUE TO OBLIQUITY OF ECLIPTIC (प्राणकलांतरं) FOR EACH DEGREE OF THE TROPICAL LONGITUDE MEASURED FROM THE NEAREST EQUINOCTIAL POINT.

(In arcual measure)

Degree	Equation		Degree	Equation		Degree	Equation		Degree	Equation		Degree	Equation	
	'	"		'	"		'	"		'	"		'	"
0	0	0	20	92	18	40	144	16	60	191	6	80	52	49
1	4	58	21	96	8	41	145	47	61	128	33	81	47	45
2	9	55	22	99	49	42	146	38	62	125	50	82	42	37
3	14	51	23	103	28	43	147	18	63	122	57	83	37	25
4	19	46	24	107	—	44	147	47	64	119	55	84	32	10
5	24	40	25	110	25	45	148	12	65	116	43	85	26	52
6	29	34	26	113	43	46	148	14	66	113	22	86	21	33
7	34	24	27	116	53	47	148	11	67	109	51	87	16	11
8	39	13	28	119	55	48	147	57	68	106	12	88	10	48
9	43	59	29	122	50	49	147	33	69	102	25	89	5	25
10	48	43	30	125	34	50	146	57	70	98	29	90	0	0
11	53	23	31	128	11	51	146	11	71	94	25			
12	57	59	32	130	40	52	145	13	72	90	14			
13	62	32	33	132	59	52	144	5	73	85	55			
14	67	1	34	135	9	54	142	46	74	81	30			
15	71	25	35	137	10	55	141	16	75	76	57			
16	75	45	36	139	0	56	139	38	76	72	19			
17	80	0	37	140	42	57	137	43	77	67	34			
18	84	9	38	142	13	58	135	41	78	62	44			
19	88	14	39	143	35	59	133	29	79	57	48			

## II. CONSIDERING THE EXCENTRICITY OF THE ORBIT.



of the star or the Dynamical mean sun, the two coinciding at Perigee and Apogee only

Neglecting the obliquity of the ecliptic let PBA be the sun's elliptical orbit B the place of true sun between the Perigee and the Apogee and C the corresponding position

At P the true sun has got the greatest velocity and will therefore shoot ahead of the mean sun the interval between them continuing to increase as long as the sun's true angular velocity exceeds its mean value This will be diminishing later, for both should coincide at Apogee,

The difference between them is only the equation of centre but the actual value is obtained by multiplying the angular interval by  $\cos \omega \sec^2 \delta$ , the factor connecting a small arc on the ecliptic with its corresponding projection on the equator

With the usual notation and as per the figure on page 58 let  $\overset{\circ}{\Gamma}B$  be the portion of the ecliptic  $\overset{\circ}{\Gamma}C$  the portion of the equator  $\angle B\overset{\circ}{\Gamma}C = \omega$ , the obliquity of the ecliptic  $\overset{\circ}{\Gamma}C$  the R A or  $a$   $\overset{\circ}{\Gamma}B$  the longitude of B, viz  $\odot$  and BC, the declination  $\delta$

If  $\angle B\overset{\circ}{\Gamma}C$  be the middle part  $\overset{\circ}{\Gamma}B$  and  $\overset{\circ}{\Gamma}C$  are the adjacents and their Napier's parts are  $(90^\circ - \omega)$ ,  $(90^\circ - \odot)$  and  $a$  respectively, than we have

sine of mid part = product of tangents of adjacents

$$(1 \text{ e}) \sin (90^\circ - \omega) = \tan (90^\circ - \odot) \times \tan a$$

$$(1 \text{ e}) \cos \omega = \cot \odot \tan a$$

$$\therefore \tan a = \cos \omega \tan \odot.$$

Now for a small increase of  $\odot$ , we have to find the change in  $a$  This could be done by a knowledge of Differential Calculus It could be readily understood by those who know that branch of Mathematics and it may be taken for granted by the rest

Differentiating both sides of the above equation with respect to a small increase of time  $dt$ , we have

$$\sec^2 a \frac{da}{dt} = \cos \omega \sec^2 \odot \cdot \frac{d\odot}{dt}$$

$$\therefore \frac{da}{dt} = \cos \omega \frac{\sec^2 \odot}{\sec^2 a} \frac{d\odot}{dt}$$

But as this is not independent of  $\odot$  or  $a$ , we proceed thus

If  $\odot$  be the middle part  $a$  and  $\delta$  become the opposites and the Napier's parts are  $(90^\circ - \odot)$ ,  $\delta$  and  $a$  respectively

$\therefore$  sine of mid part = product of cosines of opposites



At P the true sun has got the greatest velocity and will therefore shoot ahead of the mean sun the interval between them continuing to increase as long as the sun's true angular velocity exceeds its mean value This will be diminishing later for both should coincide at Apogee

The difference between them is only the equation of centre but the actual value is obtained by multiplying the angular interval by  $\cos \omega \sec^2 \delta$  the factor connecting a small arc on the ecliptic with its corresponding projection on the equator

With the usual notation and as per the figure on page 58 let  $\overset{\circ}{\Gamma}B$  be the portion of the ecliptic  $\overset{\circ}{\Gamma}C$  the portion of the equator  $\angle B\overset{\circ}{\Gamma}C = \omega$  the obliquity of the ecliptic  $\overset{\circ}{\Gamma}C$  the R A or  $a$   $\overset{\circ}{\Gamma}B$  the longitude of B viz  $\bigcirc$  and BC the declination  $\delta$

If  $\angle B\overset{\circ}{\Gamma}C$  be the middle part  $\overset{\circ}{\Gamma}B$  and  $\overset{\circ}{\Gamma}C$  are the adjacents and their Napier's parts are  $(90^\circ - \omega)$ ,  $(90^\circ - \bigcirc)$  and  $a$  respectively than we have

sine of mid part = product of tangents of adjacents

$$(1 \text{ e}) \sin (90^\circ - \omega) = \tan (90^\circ - \bigcirc) \times \tan a$$

$$(1 \text{ e}) \cos \omega = \cot \bigcirc \tan a$$

$$\tan a = \cos \omega \tan \bigcirc$$

Now for a small increase of  $\bigcirc$  we have to find the change in  $a$  This could be done by a knowledge of Differential Calculus It could be readily understood by those who know that branch of Mathematics and it may be taken for granted by the rest

Differentiating both sides of the above equation with respect to a small increase of time  $dt$ , we have

$$\sec^2 a \frac{da}{dt} = \cos \omega \sec^2 \bigcirc \frac{d\bigcirc}{dt}$$

$$\frac{da}{dt} = \cos \omega \frac{\sec^2 \bigcirc}{\sec^2 a} \frac{d\bigcirc}{dt}$$

But as this is not independent of  $\bigcirc$  or  $a$ , we proceed thus

If  $\bigcirc$  be the middle part  $a$  and  $\delta$  become the opposites and the Napier's parts are  $(90^\circ - \bigcirc)$ ,  $\delta$  and  $a$  respectively

sine of mid part = product of cosines of opposites

$$\therefore \sin (90^\circ - \odot) = \cos a \cos \delta$$

$$\therefore \cos \odot = \cos a \cos \delta$$

$$\therefore \sec \odot = \sec \delta \sec a$$

$$\therefore \frac{\sec \odot}{\sec a} = \sec \delta$$

Substituting this value for  $\frac{\sec \odot}{\sec a}$ , we get,

$$\frac{da}{dt} = \cos \omega \sec^2 \delta \frac{d\odot}{dt}$$

(1 e)  $\cos \omega \sec^2 \delta$  is the factor by which change in longitude is to be multiplied to get the corresponding change in the R A

Since the multiplier varies with the declination it follows that the greatest equation of time due to excentricity will not correspond to the greatest equation of centre. Besides as the apse line is moving forward with respect to the moving first point of Aries annually by  $61''47$ , the same equation of centre will not correspond with the same declination

But for practical purposes the equation of centre may be taken as the equation of time due to ellipticity. This is additive between perigee and apogee measured in the positive direction and negative otherwise (1 e) it is positive or negative as the equation of centre

Combining the two parts, we have as equation of time equation of time due to inclination (प्राणकलांतरं) plus equation of time due to ellipticity (रविमंदफलं). This is called the real equation of time or उदयांतर संस्कारं.

The western method of calculating the planetary positions at a given instant, depends only on the mean time of the instant at the locality. But the Hindu does not mark his day with any such fictitious sun, but at the actual sunrise time and measures all subsequent intervals as so many ghatikas etc as having passed from the true sunrise

Thus the necessity of the प्राणकलांतरं and the चरफलं which will be explained in the following chapter is to a very great extent for the Hindus to calculate the planetary positions correctly at a given instant and place. These have been explained at length to give the readers a thorough idea of the comparative methods of the Hindus and those of the modern astronomers

# Chapter IX

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## POSITION OF THE OBSERVER—CORRECTIONS DUE TO.

The position of the observer on the surface of the earth has to be fixed first of all, with respect to its latitude and longitude. These require no explanation as they are merely the usual geographical terms. Places north of the equator are reckoned as North or positive Latitude and those to the south of the equator, South or negative latitude.

The longitude is the east or the west direction from an arbitrarily chosen place — arbitrary for different authors but fixed for that particular mode of calculation. At present Greenwich has been taken as the zero meridian (Geographical) from which longitudes east or west are reckoned. We propose to take Tanjore for the mode of calculation of this work.

Places east of Tanjore (Latitude  $10^{\circ}$ – $47^{\circ}$  N and longitude  $79^{\circ}$ – $15^{\circ}$  E of Greenwich) will have their local times more or additive and those west of the place chosen less or subtractive as compared with the local time at Tanjore at a particular instant.

The planetary positions given at the beginning of the VI Chapter have been calculated for the mean sunrise at Tanjore on 1st January 1800 and the corrections to the planetary positions due to this cause will have to be applied to reversely that is if the desired place is east of Tanjore the corrections will be negative and if west positive.

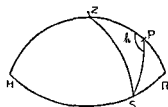
## CORRECTION DUE TO LATITUDE.

This correction has to be made for the reason that at different latitudes on the surface of the earth the sun remains above the visible horizon for different periods of time and the duration of day is thus different at different places. Also, the mean duration of day is for 12 hours (i.e.) 6 hours from sunrise to noon and 6 hours from noon to sunset. Therefore a correction due to the difference in length of the normal day of 12 hours and that of the day in question at a specified place is essential and this is called the Charaphalam (चरफले). In other words it is the difference between the hourangle of sun at sunrise and  $90^{\circ}$ . The difference in degrees converted

to hours @  $15^\circ$  for an hour when added to or subtracted from 6 hrs according as the hourangle at sunrise is greater or less than  $90^\circ$  will give the duration of half the day

## TO FIND THE HOURANGLE OF SUN AT THE TIME OF SUNRISE.

With the usual notation let HZR be the meridian of the place and HSR be the horizon S is the position of the sun or body at its rising P the celestial pole Then the angle ZPS is the hourangle at its rising



PR is the latitude  $\phi$  of the place

PS is the North polar distance of the sun and equal to  $(90^\circ - \delta)$

$$\angle SPR = \text{supplement of } \angle ZPS \\ = (180^\circ - h)$$

Now since Z is the pole corresponding to the section HSR of the celestial sphere  $\angle ZRS$  is  $90^\circ$  and hence  $\triangle SPR$  is a *rt angled* Spherical triangle of which the sides SP and PR are known and  $\angle SPR$  could be found out by applying Napier's analogies

If  $\angle SPR$  is taken as the middle part SP and RP are its adjacents but the Napier's parts of SP  $\angle SPR$  and PR are respectively  $(90^\circ - SP)$   $(90^\circ - \angle SPR)$  and PR (i.e.)  $(90^\circ - 90^\circ - \delta)$ ,  $(90^\circ - 180^\circ - h)$  and  $\phi$  respectively (i.e.)  $\delta$ ,  $(h - 90^\circ)$  and  $\phi$  respectively

Sine of middle part = the product of the tangents of adjacents

$$(i.e.) \sin (h - 90^\circ) = \tan \delta \times \tan \phi$$

$$(i.e.) -\cos h = \tan \phi \tan \delta$$

$$(i.e.) \cos h = -\tan \phi \tan \delta$$

This is a very important formula for it explains how the duration of day varies with the latitude and declination

So long as the declination is positive which is the case when the sun's tropical longitude is between 0 to  $180^\circ$  for places with the North latitude  $\cos h$  is negative (i.e.)  $h$  will be greater than  $90^\circ$ , indicating thereby that the duration of day is more than the mean duration of 12 hrs

For negative or South declination for those same places with North latitude  $\cos h$  is positive indicating thereby  $h$  to be less than  $90^\circ$  or that the duration of the day is less than 12 hours



Similarly the conditions are reversed for south latitudes

The formula  $\cos h = -\tan \phi \tan \delta$ , is thus depending on the declination which in turn depends on the tropical longitude of the sun. It will be convenient to have the same directly as an explicit function of the longitude. This is done as follows

With the usual notation in the figure of page, 58 if  $\delta$  is taken as the middle part,  $\odot$  and  $\omega$  are the opposites, of which the Napier's parts are tropical  $(90^\circ - \odot)$  and  $(90^\circ - \omega)$ .

$$\begin{aligned}\therefore \sin \delta &= \cos (90^\circ - \odot) \cos (90^\circ - \omega) \\ &= \sin \odot \sin \omega\end{aligned}$$

$\therefore$  Substituting this value of  $\sin \delta$  in the previous equation, we get

$$\cos h = \frac{\tan \phi \sin \omega \sin \odot}{\sqrt{1 - \sin^2 \omega \sin^2 \odot}}$$

This formula gives the hourangle required directly from the tropical longitude of the sun

The excess of the hourangle thus arrived at, over  $90^\circ$  or its defect from  $90^\circ$ , gives the चरफलं. It is positive between  $0^\circ$  to  $180^\circ$  of tropical longitude and negative between  $180^\circ$  to  $360^\circ$  of the same for purposes of determination of duration of day

The cases of long duration of day will make the sunrise earlier than 6 A M and set later than 6 P M and those of short duration will make the sunrise later than 6 A M and set earlier than 6 P M. Hence when the चरफलं has to be applied to the apparent sunrise time to correct it to mean sunrise time, the चरफलं positive or negative as derived by the method laid out, will be considered with reversed signs for purposes of applying to the planetary positions. This should be clearly understood

There is no charaphalam for  $0^\circ$  of latitude for  $\cos h = -\tan \phi \tan \delta$ , when  $\phi = 0$ , reduces to 0 independent of the value of  $\delta$ . Therefore at the equator,  $\cos h = 0$ , (i.e.)  $h = 90^\circ$  or the duration of day is always 12 hrs

## DETERMINATION OF LATITUDE OF A PLACE.

For the determination of latitude by really scientific methods a text book of Mathematical Astronomy may be consulted. Various methods of determination are given based on the directly observed positions of heavenly

Let us now arrive at the values of चरफले for  $30^\circ$ ,  $60^\circ$  and  $90^\circ$  by purely trigonometrical method and compare them with the previous values as arrived at by the Hindu method

### FOR $30^\circ$ OF TROPICAL LONGITUDE.

$$\cos h = \frac{-\tan \phi \sin \omega \sin \odot}{\sqrt{1 - \sin^2 \omega \sin^2 \odot}}$$

$$\phi = 10^\circ - 47' \text{ N}, \omega = 23^\circ - 27\frac{1}{2}', \odot = 30^\circ$$

$$\tan \phi = \tan 10^\circ - 47' \text{ N} = .1904587$$

$$\sin \omega = \sin 23^\circ - 27\frac{1}{2}' = .3980821$$

$$\sin \odot = \sin 30^\circ = .5$$

$$\begin{aligned} \therefore \cos h &= \frac{-1904587 \times .3980821 \times .5}{\sqrt{1 - (.3980821 \times .5)^2}} \\ &= \frac{-1904587 \times 19904105}{\sqrt{1 - 19904105 \times 80095895}} \\ &= \frac{-.03790909}{.9799912} \\ &= -.0386831 = \cos (92^\circ - 13' - 1'') \end{aligned}$$

Since  $\cos h$  is negative,  $h$  should be in the II quadrant. (vide explanation under the following para)

$$\therefore h = 92^\circ - 13' - 1''$$

$$\begin{aligned} \therefore \text{प्रथमचरखंडे, or चरफले (charaphalam) is } 92^\circ - 13' - 1'' \text{ minus } 90^\circ \\ = 2^\circ - 13' - 1'' \text{ or } .133^\circ - 1'' \end{aligned}$$

### FOR $60^\circ$ OF TROPICAL LONGITUDE.

Similarly for  $60^\circ$  of tropical longitude, all the others remain the same in the formula except  $\sin 60^\circ$  which is .8660254 has to be substituted

$$\begin{aligned} \cos h &= \frac{-1904587 \times .3980821 \times .8660254}{\sqrt{1 - (.3980821 \times .8660254)^2}} \\ &= \frac{-.065660642}{\sqrt{.83114799}} \\ &= \frac{-.065660642}{.9116748} = -.06994887 \end{aligned}$$

$\cos 94^\circ - 0 - 40 = -0.6994887$  for as the cosine is negative the argument should be in the II quadrant and not in the III quadrant though the cosine in the III quadrant is also negative for the matter that the  $h$  with which we are concerned cannot be very widely separated from  $90^\circ$  which will be the case if the angle in the III quadrant be taken

• Charaphalam at  $60^\circ$  of the tropical longitude is  $4^\circ - 0 - 40''$   
or  $240' - 40''$

$$\begin{aligned} \therefore \text{द्वितीयचरखंड} &= 240' - 40'' \text{ minus} \\ &\quad \underline{133' - 1''} \\ &= \underline{107' - 39''} \end{aligned}$$

### FOR $90^\circ$ OF THE TROPICAL LONGITUDE.

$$\begin{aligned} \cos h &= \frac{-1904587 \times 3980821 \times 1}{\sqrt{1 - (3980821 \times 1)^2}}, \\ \text{for } \sin 90^\circ &= 1 \\ \cos h &= \frac{-0.7581838}{\sqrt{1 - 3980821 \times 6019179}} \\ &= \frac{-0.7581838}{0.9173497} = -0.826494 \end{aligned}$$

$$\underline{\underline{= 43' - 47''}}$$

Summing up we have

	Trigonometrical method	Hindu method
I portion	133 - 1"	137 - 7" 8
II portion	107 - 39"	109 - 42" 24
III portion	43 - 47"	45 - 42" 6
Total	<u>284 - 27"</u>	<u>294 - 32" 64</u>

From a comparison of the two sets of values it will be seen that there is difference between each corresponding portion and also in the total

the difference being due to the maximum declination having been taken as  $24^\circ$  by Hindus instead of the mean value  $23^\circ-27\frac{1}{2}$ . Yet the difference in the maximum charaphalams is only 10 of arc or less than 2 vighatis of time which is highly commendable for the nearness of the two results

These tedious yet interesting calculations are not done in vain and their use will be seen as we proceed. Considering the labouriousness of these workings the annexed table could be used to find out the (चरफल) Charaphalam for a desired tropical longitude taking care to prefix the proper sign with due regard to the quadrant the tropical longitude lies in

## TABLE OF CHARAPHALAM FOR EACH DEGREE OF DISTANCE OF TROPICAL LONGITUDE MEASURED FROM THE NEAREST EQUINOX.

Calculated for Tanjore  $10-47^\circ$  N

the difference being due to the maximum declination having been taken as  $24^{\circ}$  by Hindus instead of the mean value  $23^{\circ}-27\frac{1}{2}$ . Yet the difference in the maximum charaphalam is only 10 of arc or less than 2 vighatis of time which is highly commendable for the nearness of the two results

These tedious, yet interesting calculations are not done in vain and their use will be seen as we proceed. Considering the labouriousness of these workings the annexed table could be used to find out the (चरफलं) Charaphalam for a desired tropical longitude, taking care to prefix the proper sign with due regard to the quadrant the tropical longitude lies in

### TABLE OF CHARAPHALAM FOR EACH DEGREE OF DISTANCE OF TROPICAL LONGITUDE MEASURED FROM THE NEAREST EQUINOX.

Calculated for Tanjore  $10^{\circ}-47^{\circ}$  N

As indicated on the top of the tables of charaphalam, the same has been calculated for Tanjore, whose latitude is  $10^{\circ}-47' N$ . If it be required to know the corresponding figures for any other latitude proceed as follows

If the tropical longitude is within  $180^{\circ}$ , (i.e.) in the I and II quadrants, increase  $90^{\circ}$  by the charaphalam. Find the cosine of the angle thus arrived at. Multiply it by the tangent of latitude of the given place and divide it by the tangent of the latitude of Tanjore—viz  $10^{\circ}-47'$ . Note the result. Find whose cosine is the result. That angle less  $90^{\circ}$  will be the required charaphalam.

In the case of III and IV quadrants, diminish  $90^{\circ}$  by the charaphalam. Find the cosine of the angle as before and multiply it by the ratios of the tangents of the latitude of the given place and that of Tanjore. Find the angle whose cosine is the result. The difference between  $90^{\circ}$  and the resultant angle will be the required charaphalam.

The method of merely multiplying the charaphalam by the ratio of the tangents is very rough and may fail to give even an approximate value beyond say  $20^{\circ}$  or so. A passing mention guarding against this risky, though short cut, has already been made in the previous pages.

Reverting to the example taken tropical longitude of sun

= Longitude of sun + precession

=  $2^{\circ}-29'-21''-18''$  plus  $0^{\circ}-22'-36''-48''$

Example, net result is  $2^{\circ}-26''$

sun's true velocity  $57'-13''$  (फट्टरवि).

Correction to be applied =  $\frac{28 \times 13}{21600} = 4$

This is positive as the net result is also positive. Hence adding it to the true longitude of sun already found viz  $2^{\circ}-29'-21''-18''$ , we get  $2^{\circ}-29'-21''-22''$ .

This is the rectified true longitude of sun

## Chapter X

### POSITION OF THE ECLIPTIC.

If the equator were a bright visible band in the sky, it would occupy a fixed position of which we may readily obtain an idea by means of some of its points namely, the East point with its opposite the west point and a

The mean longitude of sun at the time of birth is  $2^{\circ}-29'-12''-10''$  referred to Tanjore. Place of birth is 255 west of Tanjore. Correction due to this is 42' of arc (plus) to be added to the mean longitude of sun referred to above. [255' of longitudinal difference is approximately  $1'' = 12.5$  vighatikas of time. As sun's mean velocity is  $59'-8''$  for a day, correction due to 42.5 vighatikas is nearly 42' of arc. This is positive as the longitudinal difference is additive for time but subtractive for planets as we proceed east of a chosen point and *vice-versa* when proceeding to the west].

$\therefore$  Correct mean longitude of sun at birth at the place of birth is  $2^{\circ}-29'-18''-22''$ . If we add the precession  $22^{\circ}-36'-45''$  to this we get  $3^{\circ}-22^{\circ}-20'-10''$  as the tropical longitude of mean sun.

Time of birth is 2 A.M. Indian standard time. Therefore local time of birth is 2 A.M. minus 30.6 mts which is 1 hr 29.4 mts A.M.

\* Time passed since the previous noon is 12 hrs + (1 hr - 29.4 mts) = 13 hrs - 29 mts - 24 sec.

Converting it into degree etc.  $\therefore 15$  per hr we get  $202^{\circ}-21'-0''$ . Adding this to the mean tropical longitude of sun we get  $8^{\circ}-22^{\circ}-20'-10''$  (or  $112^{\circ}-20'-10''$ ) plus  $202^{\circ}-21'-0''$ , which is equal to  $314^{\circ}-41'-10''$ . This is called the Right ascension of midheaven or R.A.M.C. as it is usually known among astronomers. It is with this R.A.M.C. we proceed to find out the ascending point and the culminating point.

## ASCENDING POINT.

We shall take the formula previously derived which gives the point N from which we can get the ascending point by merely adding 90.

$$\tan \gamma N = \frac{\tan \phi \sin \omega}{\cos M} + \cos \omega \tan M$$

$$\phi = 12^{\circ}-45' N \quad \omega = 23^{\circ}-27\frac{1}{2}' \quad \text{and} \quad M = 314^{\circ}-41'-10''$$

$$\begin{array}{l} \tan 12^{\circ}-45' N = 2262769 \\ \sin 23^{\circ}-27\frac{1}{2}' = 3980821 \\ \cos 23^{\circ}-27\frac{1}{2}' = 9173498 \\ \tan 314^{\circ}-41'-10'' = \tan (360^{\circ}-45^{\circ}-18'-50'') \\ \quad = -\tan 45^{\circ}-18'-50'' \\ \quad = -1.0110178 \end{array} \quad \left| \begin{array}{l} \cos (314^{\circ}-41'-10'') \\ = \cos (360^{\circ}-45^{\circ}-18'-50'') \\ = \cos 45^{\circ}-18'-50'' \\ = .7032224 \quad (\text{This is positive} \\ \text{being in the IV quadrant}) \end{array} \right.$$

$$\therefore \tan \gamma N = \frac{2262769 \times .3980821}{.7032224} + .9173498 \times -1.0110178$$

$$= 12809014 - 9274565$$

$$= -79936636$$

$$= \tan (-38^{\circ}-38'-16'')$$

$$\left( \begin{array}{l} \text{for } \tan 38^{\circ}-38' = 7992425 \\ \tan 38^{\circ}-39' = 7997193 \end{array} \right)$$

$$\text{or } \tan (180^{\circ}-38^{\circ}-38'-16'')$$

The angle whose tangent is negative may be either in the II or IV quadrant. But we should choose that which is nearer to the R A M C. In this case, the R A M C is in the IV quadrant. Hence the required value of 'N' will be  $(360^{\circ}-38^{\circ}-38'-16'')$  or  $321^{\circ}-21'-44''$ . Adding  $90^{\circ}$  to this we get the longitude of Ascendant as  $411^{\circ}-21'-44''$  or  $51^{\circ}-21'-44''$  after subtracting  $360^{\circ}$ .

$\therefore$  The tropical longitude of the ascendant is therefore  $51^{\circ}-21'-44''$ . The precession when subtracted from this will give the longitude of ascending point as per the Hindu Nirayana reckoning. It will be  $(51^{\circ}-21'-44'')$  minus  $(22^{\circ}-36'-48'')$ , which is equal to  $28^{\circ}-44'-56''$ .

$$\therefore \text{Nirayana Longitude of Ascendant} = 0^{\circ}-29^{\circ}-44' \quad 56''$$

## HINDU METHOD.

Take sayanasphuta Ravi (सायनस्फुटरवि) that is  $(2^{\circ}-29^{\circ}-21'-22'')$  plus  $(0^{\circ}-22^{\circ}-36'-48'')$  which is  $3^{\circ}-21^{\circ}-58'-10''$ . We have already worked out the चरफल and the प्राणफलंतरं as  $(311-36)$  ऋण (-) and  $106-5$  धन (+) respectively. The net result is  $205^{\circ}-31'$  ऋण (-) converting this to degrees we have  $3^{\circ}-25'-31''$  ऋण (-). Being negative subtract this from the tropical true longitude (सायनस्फुटरवि).

The result will be the उदयकाललग्न (Udayakala-lagnam). If the net result had been positive, it should have been added to the true tropical longitude.

$$\begin{array}{r} \text{We have therefore} \quad 3^{\circ}-21^{\circ}-58'-10'' \\ \text{minus} \quad 3^{\circ}-25'-31'' \end{array}$$

$$\text{This is Udayakala Lagnam} \quad 3^{\circ}-18^{\circ}-32'-39''$$

Now take the birth time in ghatikas after true sunrise (i.e.) 49 ghatikas  $-17.5$  vighaties. Multiplying this by 6 we get degrees etc. It will be  $295^{\circ}-45'-0''$ . Converting the degrees to signs etc and adding it to the Udayakala Lagnam we get the काललग्न. It is  $3^{\circ}-18^{\circ}-32'-39''$  plus  $9^{\circ}-25^{\circ}-45'-0''$

$$\text{(Kalalagnam)} = 1^{\circ}-14^{\circ}-17'-39''$$



The mean longitude of sun at the time of birth is  $2^{\circ}-29'-12''-10$  referred to Tanjore. Place of birth is 255 west of Tanjore. Correction due to this is 42 of arc (plus) to be added to the mean longitude of sun referred to above. 255 of longitudinal difference is approximately  $12\frac{1}{2}$  vighatikas of time. As sun's mean velocity is  $59'-6''$  for a day correction due to 42.5 vighatikas is nearly 42 of arc. This is positive as the longitudinal difference is additive for time but subtractive for planets as we proceed east of a chosen point and *vice-versa* when proceeding to the west.

$\therefore$  Correct mean longitude of sun at birth at the place of birth is  $2^{\circ}-29'-41''-22''$ . If we add the precession  $22'-41''-15''$  to this we get  $3^{\circ}-22'-20'-10''$  as the tropical longitude of mean sun.

Time of birth is 2 A.M. Indian standard time. Therefore local time of birth is 2 A.M. minus 30.6 mts. which is 1 hr. 29.4 mts. A.M.

$\therefore$  Time passed since the previous noon is 12 hrs. + (1 hr. - 29.4 mts.) = 13 hrs. - 29 mts. - 24 sec.

Converting it into degree etc.  $\therefore 15$  per hr. we get  $202^{\circ}-21'-0''$ . Adding this to the mean tropical longitude of sun we get  $3^{\circ}-22'-20'-10''$  (or  $112^{\circ}-20'-10''$ ) plus  $202^{\circ}-21'-0''$ , which is equal to  $114^{\circ}-11'-10''$ . This is called the Right ascension of midheaven or R.A.M.C. as it is usually known among astronomers. It is with this R.A.M.C. we proceed to find out the ascending point and the culminating point.

## ASCENDING POINT.

We shall take the formula previously derived which gives the point N from which we can get the ascending point by merely adding 90.

$$\tan \gamma N = \frac{\tan \phi \sin \omega}{\cos M} + \cos \omega \tan M$$

$$\phi = 12^{\circ}-45'N \quad \omega = 23^{\circ}-27\frac{1}{2}' \quad \text{and} \quad M = 314^{\circ}-41'-10''$$

$$\begin{array}{l} \tan 12^{\circ}-45'N = 2262769 \\ \sin 23^{\circ}-27\frac{1}{2}' = 3980821 \\ \cos 23^{\circ}-27\frac{1}{2}' = 9173498 \end{array} \quad \left| \begin{array}{l} \cos (314^{\circ}-41'-10'') \\ = \cos (360^{\circ}-15^{\circ}-18'-50'') \\ = \cos 45^{\circ}-18'-50'' \end{array} \right.$$

$$\begin{array}{l} \tan 314^{\circ}-41'-10'' = \tan (360^{\circ}-45^{\circ}-18'-50'') \\ = -\tan 45^{\circ}-18'-50'' \\ = -1.0110173 \end{array} \quad \left| \begin{array}{l} = 7032221 \text{ (This is positive} \\ \text{being in the IV quadrant)} \end{array} \right.$$

$$\therefore \tan \gamma N = \frac{2262769 \times 3980821}{7032221} + 9173498 \times -1.0110173$$

$$= .12609014 - .9274565$$

$$= -.79936636 \quad = \tan (-38^{\circ}-38'-16'')$$

$$\left( \begin{array}{l} \text{for } \tan 38^{\circ}-38' = .7992425 \\ \tan 38^{\circ}-39' = .7997193 \end{array} \right) \quad \text{or } \tan (180^{\circ}-38^{\circ}-38'-16'')$$

The angle whose tangent is negative may be either in the II or IV quadrant. But we should choose that which is nearer to the R A M C. In this case, the R A M C is in the IV quadrant. Hence the required value of 'N will be  $(360^{\circ}-38^{\circ}-38'-16'')$  or  $321^{\circ}-21'-44''$ . Adding  $90^{\circ}$  to this, we get the longitude of Ascendant as  $411^{\circ}-21'-44''$  or  $51^{\circ}-21'-44''$  after subtracting  $360^{\circ}$ .

$\therefore$  The tropical longitude of the ascendant is therefore  $51^{\circ}-21'-44''$ . The precession when subtracted from this will give the longitude of ascending point as per the Hindu Nirayana reckoning. It will be  $(51^{\circ}-21'-44'')$  minus  $(22^{\circ}-36'-48'')$ , which is equal to  $28^{\circ}-44'-56''$ .

$$\therefore \text{Nirayana Longitude of Ascendant} = 0^{\circ}-28^{\circ}-44' \quad 56''$$

## HINDU METHOD.

Take sayanasphuta Ravi (सायनस्फुटरवि) that is  $(2^{\circ}-29^{\circ}-21' -22')$  plus  $(0^{\circ}-22^{\circ}-36'-48')$  which is  $3^{\circ}-21^{\circ}-58'-10'$ . We have already worked out the चरफलं and the प्राणकलांतरं as  $(311-36)$  ऋणं  $(-)$  and  $106-5$  धनं  $(+)$  respectively. The net result is  $205^{\circ}-31'$  ऋणं  $(-)$ . Converting this to degrees we have  $3^{\circ}-25'-31''$  ऋणं  $(-)$ . Being negative subtract this from the tropical true longitude (सायनस्फुटरवि.).

The result will be the उदयकाललग्नं (Udayakala-lagnam). If the net result had been positive, it should have been added to the true tropical longitude.

$$\begin{array}{r} \text{We have therefore} \quad 3^{\circ}-21^{\circ}-58'-10'' \\ \text{minus} \quad 3^{\circ}-25'-31'' \end{array}$$

$$\text{This is Udayakala Lagnam} \quad 3^{\circ}-18^{\circ}-32'-39''$$

Now take the birth time in ghatikas after true sunrise (i.e.) 49 ghatikas  $-175$  vighatias. Multiplying this by 6 we get degrees etc. It will be  $295^{\circ}-45'-0''$ . Converting the degrees to signs etc and adding it to the Udayakala Lagnam we get the काललग्नं. It is  $3^{\circ}-18^{\circ}-32'-39''$  plus  $9^{\circ}-25^{\circ}-45'-0''$

$$\text{(Kalalagnam)} = 1^{\circ}-14^{\circ}-17'-39''$$

With this Kalalagnam we find out the charaphalam (चरं) and (प्राण) Equation due to obliquity and apply the net result to the काललग्नं itself *with reversed signs*. The process is repeated until two consecutive results agree. This is what is known as स्थिरीकरणं corresponding to 'Successive approximation' and the result got thereby is called the चरप्राणकलांतर स्थिरीकृतसायनलग्न. The result of the last trial subtracted by the precession will be the Nirayana longitude of Ascendant.

We shall do by the Hindu method the same calculation of finding out the ascendant.

# I TRIAL

Kalalagnam काललग्नं

1

14

17

39

or 44°-17-39

As this is within 90 the distance from the nearest equinox is the same viz 44-17-39

for 44° चरं is 186'-27" प्राण is 147-47

for 45° चरं is 190'-2" प्राण is 148-12

∴ for 44°-17-39" चरं is 187-30" प्राण is 147-54"

As the tropical longitude is within 180° and also in the I quadrant चरं is negative and प्राण also is negative

The चरं found out here is for Tanjore but we want for Mangalore. Multiplying it by the ratio of the tangents of the latitudes of the two places viz 1879 we get चरं for the place of birth as 222-43 (Though multiplying by the ratio of the tangents of the latitudes has been stated to be only very rough it has been resorted to only as an example. A tabular statement with instruction to work out a table of चरं for any desired latitude is appended in the appendix whereby there will be no need to resort to this rough way.)

∴ Rectified चरं is 222'-48" ऋणं (—)

प्राण is 147-54" ऋणं (—)

Net result is 370'-37" ऋणं (—)

or 6-10-37" ऋणं

Applying this reversedly to काललग्नं, we get 1°-14'-17-39" plus 6°-10'-37"

1°-20'-26'-16"

This is a first approximation

## II TRIAL

1  
20  
28  
16  
or 50°—28'—16"

As this is within 90°, the same is the distance from the nearest equinox

for 50° चरं is 207'—38' प्राणं is 146'—57'  
for 51° चरं is 211'—4' प्राणं is 146'—11"

∴ for 50°—28'—16" चरं is 209—15' and प्राणं is 146'—35'

The former is negative as also the latter, for the II trial tropical longitude is still less than 180° and in the I quadrant

Rectified चरं for the place of birth

$$= 209'—15'' \times 1.1879 = 247'—31'' \text{ ऋणं}$$

$$\text{प्राणं} = 146'—35'' \text{ ऋणं}$$

$$\begin{aligned} \text{Net result} &= 394'—6'' \text{ ऋणं} \\ &= 6°—34'—6'' \text{ ऋणं} \end{aligned}$$

Applying this reversedly to the original Kalalagmam 1°—14'—17'—39", we get 1°—20°—51'—45". This is a II approximation

## III TRIAL

1  
20  
51  
45  
or 50°—51'—45"

for 50° चरं is 207'—38" and प्राणं is 146'—57"

for 51° चरं is 211'——1" and प्राणं is 146'—11"

∴ for 50°—51'—45' चरं is 210'—36" and प्राणं is 146'—17".

As before चरं is ऋणं (—) and प्राणं also is ऋणं (—). Rectified चरं for the place of birth

$$\begin{aligned} &= 210'—36'' \times 1.1879 = 250'—10'' \text{ ऋणं} \\ \text{प्राणं} &= 146'—17'' \text{ ऋणं} \end{aligned}$$

$$\begin{aligned} \text{Net result is} &= 396'—27'' \text{ ऋणं} \\ &\text{or } 6°—36'—27'' \text{ ऋणं} \end{aligned}$$

Applying this reversedly to the original Kalalagmam, we get 1°—20°—51'—6" as the III approximation

With this Kalalagnam we find out the charaphalam (चरं) and (प्राण) Equation due to obliquity and apply the net result to the कालग्नं itself *with reversed signs*. The process is repeated until two consecutive results agree. This is what is known as स्थिरीकरणं corresponding to 'Successive approximation' and the result got thereby is called the चरप्राणकलांतर स्थिरीकृतसायनग्नं. The result of the last trial subtracted by the precession will be the Nirayana longitude of Ascendant.

We shall do by the Hindu method the same calculation of finding out the ascendant.

# 1 TRIAL

Kalalagnam कालग्नं

1

14

17

39

or  $44^{\circ}-17'-39''$

As this is within 90 the distance from the nearest equinox is the same viz  $44^{\circ}-17'-39''$

for  $44^{\circ}$  चरं is  $186'-27''$  प्राण is  $147'-47''$

for  $45^{\circ}$  चर is  $190'-2''$  प्राण is  $148'-12''$

$\therefore$  for  $44^{\circ}-17'-39''$  चरं is  $187'-30''$  प्राणं is  $147'-54''$

As the tropical longitude is within  $180^{\circ}$  and also in the I quadrant चरं is negative and प्राणं also is negative

The चरं found out here is for Tanjore but we want for Mangalore. Multiplying it by the ratio of the tangents of the latitudes of the two places viz 1879 we get चर for the place of birth as  $222'-43''$  (Though multiplying by the ratio of the tangents of the latitudes has been stated to be only very rough it has been resorted to only, as an example. A tabular statement with instruction to work out a table of चर for any desired latitude is appended in the appendix whereby there will be no need to resort to this rough way.)

$\therefore$  Rectified चरं is  $222'-43''$  ऋण (—)

प्राणं is  $147'-54''$  ऋणं (—)

Net result is  $370'-37''$  ऋणं (—)

or  $6^{\circ}-10'-37''$  ऋणं

Applying this reversedly to कालग्नं, we get  $1^{\circ}-14^{\circ}-17'-39''$  plus

$6^{\circ}-10'-37''$

$1^{\circ}-20^{\circ}-28'-16''$

This is a first approximation

## II TRIAL

1  
20  
28  
16  
or 50—28—16'

As this is within 90 the same is the distance from the nearest equinox

for 50 चरं is 207—38 प्राणं is 146—57

for 51° चरं is 211—4" प्राणं is 146—11"

∴ for 50°—28'—16" चरं is 209—15 and प्राणं is 146—35

The former is negative as also the latter for the II trial tropical longitude is still less than 180 and in the I quadrant

Rectified चरं for the place of birth

$$= 209'—15'' \times 1.1679 = 247'—31'' \text{ ऋणं}$$

$$\text{प्राणं} = 146'—35'' \text{ ऋणं}$$

$$\begin{array}{rcl} \text{Net result} & = & 394'—6'' \text{ ऋणं} \\ & = & 6^\circ—34'—6'' \text{ ऋणं} \end{array}$$

Applying this reversedly to the original Kalalagnam  $1^\circ—14'—17''$  30, we get  $1^\circ—20'—51'—45''$ . This is a II approximation

## III TRIAL

1  
20  
51  
45  
or 50°—51'—45"

for 50° चरं is 207'—38" and प्राणं is 146—57"

for 51° चरं is 211'—4" and प्राणं is 146—11"

∴ for 50°—51'—45" चरं is 210'—36" and प्राणं is 146'—17".

As before चरं is ऋणं (—) and प्राणं also is ऋणं (—). Rectified चरं for the place of birth  $= 210'—36'' \times 1.1679 = 250'—10''$  ऋणं

$$\text{प्राणं} = 146'—17'' \text{ ऋणं}$$

$$\text{Net result is} = 396—27'' \text{ ऋणं}$$

$$\text{or } 6^\circ—36'—27'' \text{ ऋणं}$$

Applying this reversedly to the original Kalalagnam we get  $1^\circ—20'—51'—6''$  as the III approximation

It could have been seen that the II trial gave a large difference from the I and that the III trial gave only a small difference from that of the II. If a IV trial also were made, the result would be practically the same as that of the III trial. Thus we can stop with the III trial and take it as the final result. Subtracting therefrom the precession, we get the ascendant of the moment.

In this case, the final result of the last trial is  $1^s-20^{\circ}-54'-6''$ . Subtracting precession  $22^{\circ}-36'-48''$ , we get  $0^s-28^{\circ}-17'-18''$ . This is the Nirayana longitude of the ascendant according to the Hindu method. There is a difference between this final result and that arrived at by the trigonometrical method the difference is due to our having used the multiplier ratio of the tangents of latitudes, instead of employing a table of चरफलं specially made for the latitude of the place. This point has already been stressed upon and the appendix furnishes the clue to prepare a table of चरफलं for a desired place.

## TO FIND THE CULMINATING POINT OR LONGITUDE OF THE TENTH HOUSE. (TRIGONOMETRICAL METHOD)

The longitude of the tenth house or the culminating point is given by the formula,  $\tan \gamma^{\circ}C = \frac{\tan M}{\cos \omega}$ .

In the present example,  $M = 314^{\circ}-41'-10''$  and  $\omega = 23^{\circ}-27\frac{1}{2}'$

$$\therefore \tan \gamma^{\circ}C = \frac{\tan 314^{\circ}-41'-10''}{\cos 23^{\circ}-27\frac{1}{2}'}$$

$$\begin{aligned} \text{now, } \tan 314^{\circ}-41'-10'' &= \tan (360^{\circ}-45^{\circ}-18'-50'') \\ &= -\tan 45^{\circ}-18'-50'' \\ &= -1.0110173 \end{aligned}$$

$$\text{and } \cos 23^{\circ}-27\frac{1}{2}' = 0.9173498$$

$$\therefore \tan \gamma^{\circ}C = \frac{-1.0110173}{0.9173498} = -1.1021007$$

$$= \tan (-47^{\circ}-46'-51'')$$

$$\text{or } \tan (180^{\circ}-47^{\circ}-46'-51'')$$

For, an angle whose tangent is negative may be either in the IV or II quadrant. Here the value in the IV quadrant only has to be taken being nearer to the R A M C than the other.

$$\therefore 7^{\circ}\text{C} = -47^{\circ}-46'-51'' \text{ or } 360^{\circ}-47^{\circ}-46'-51'' \\ = 312^{\circ}-13'-9''$$

This includes precession of  $22^{\circ}-36'-48''$ , subtracting it we have longitude of midheaven as  $289^{\circ}-36'-21''$  or  $9^{\circ}-19'-36'-21''$ .

## TENTH HOUSE BY HINDU METHOD.

Take the tropical longitude of the ascendant as arrived at by the last trial It is  $1^{\circ}-20'-54'-6''$  From this obtain चरफल and प्राणकलांतर.

1				This is within $90^{\circ}$ and hence it itself is the
20		50"		argument for referring to the tables of चरफल
54	or	54		(Charaphalam) and प्राणकलांतर (Equation of time
6		6		due to obliquity)

for 50th item चर is  $207'-38''$  प्राण is  $146'-57''$

51st item चर is  $211'-4''$  प्राण is  $146'-11''$

$\therefore$  For  $50-54-6''$  चर is  $210'-44''$  प्राण is  $146'-16''$

both are negative (ऋण) as the tropical longitude of the ascendant taken is less than  $180^{\circ}$  and also in the I quadrant चरफल for the desired latitude is  $210'-44'' \times 1.1879 = 250'-20''$  ऋण (—).

$\therefore$  Net result of चरफल and प्राणकलांतर is  $-(396'-16'')$  or  $-(6^{\circ}-36'-16'')$

This is applied directly to the tropical longitude  $1^{\circ}-20'-54'-6''$  We get  $1^{\circ}-14'-17'-50''$  With this again प्राणकलांतर is found out The same is the argument being less than  $90^{\circ}$

For  $44^{\circ}$  प्राण is  $147-47$  and for  $45^{\circ}$  it is  $148-12$

$\therefore$  For  $44^{\circ}-17'-50''$  it is  $147-54$  ऋण (—), as the tropical longitude  $1^{\circ}-14'-17'-50''$  is in the I quadrant

Combined चरफल and प्राणकलांतर is  $6^{\circ}-36'-16''$  ऋण  
the present प्राणकलांतर is  $2^{\circ}-27'-54''$  ऋण

Net result is  $9^{\circ}-4'-10''$  ऋण

Being ऋण, subtract this from the original Nirayana longitude of the ascendant and also 3 signs always, when we get the Nirayana longitude of the Mid-heaven or the tenth house



Nirayana Lagnasphutam is  $0^{\circ}-28^{\circ}-17'-18''$

Net result found out is  $9^{\circ}-4'-10''$  ऋणं

Subtracting we get  $0^{\circ}-19^{\circ}-13'-8''$

Subtracting also 3 signs  $3^{\circ}-0^{\circ}-0'-0''$

Longitude of 10th house  $9^{\circ}-19^{\circ}-13'-8''$

This almost works up to the result arrived at by the trigonometrical method. It would have been still nearer had the चरफले for Mangalore calculated for that place been used instead of the multiplier ratio. But I have already stated that this is only an example giving out the method employed but for followers a method to calculate and have a ready-made table for चरफले for any desired latitude is given in the appendix.

## CALCULATION OF OTHER HOUSES BY HINDU METHOD.

Write out all the twelve houses adding one sign successively to the Nirayana longitude of the ascendant

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
°	0	1	2	3	4	5	6	7	8	9	10	11
'	28	28	28	28	28	28	28	28	28	28	28	28
'	17	17	17	17	17	17	17	17	17	17	17	17
''	18	18	18	18	18	18	18	18	18	18	18	18

Take the net result worked out in the determination of the tenth house, viz  $9^{\circ}-4'-10''$  ऋणं. Find one-third viz  $3^{\circ}-1'-23''$  (ऋणं) and two-thirds viz  $6^{\circ}-2'-47''$  (ऋणं) of the same.

The net result as also its  $\frac{1}{3}$  and  $\frac{2}{3}$  being ऋणं (negative), subtract net result from the IV and X houses,  $\frac{1}{3}$  from the II VI VIII and XII houses, and lastly  $\frac{2}{3}$  from the III V, IX and XI houses. The figure for the VII house is obtained by adding 6 signs to the ascendant. We get them as follows and these are called the longitudes of the houses (भाक्कुटाः)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
s	0	1	2	3	4	5	6	7	8	9	10	11
°	28	25	22	19	22	25	28	25	22	19	22	25
'	17	15	14	13	14	15	17	15	14	13	14	15
"	18	55	31	8	31	55	18	55	31	8	31	55

The Hindus consider that these House longitudes are the middle of the houses and not the cusps as the moderners do. For, the tenth house has been defined as the midheaven and hence symmetrically situated about the meridian. In other words the culminating point is only the middle of the tenth house. In keeping with this other house longitudes arranged above are only their midpoints and not the cusps.

Then to find the point where a particular house begins and where it ends, they add the longitude of the house with that of the preceeding house and the succeeding house separately and the mean of the two sums will give respectively the beginning and ending of the house taken.

### (भावसंशयः) ENDING POINTS OF EACH HOUSE.

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
s	1	2	3	4	5	6	7	8	9	10	11	0
°	11	8	5	5	6	11	11	8	5	3	8	11
'	46	45	43	43	43	46	46	45	43	43	45	46
"	37	18	50	50	18	37	37	18	50	50	18	37

As the ending points are given the beginning of each house will be the ending of the previous house.

### TRIGONOMETRICAL METHOD OF FINDING OUT THE LONGITUDES OF THE HOUSES.

The principle contained in the Hindu method of finding out the longitudes of the houses is the trisection of the semi-arc diurnal and nocturnal while that in the modern use and of common acceptance is the trisection of semi-arc of each degree of ecliptic  $\frac{1}{3}$ ,  $\frac{1}{4}$  and whole of the semi-arc (diurnal) of any degree is successively added to the sidereal time of its ascension and

• Oblique ascension of 11th house	814°—41'—10"
∴ R. A of 11th house is	254°—41'—10"

(It is usual to say that the oblique ascension of any house is 90° plus the Right ascension)

$$\begin{aligned}
 &\therefore -\cot (\text{Longitude of 11th house cusp}) \\
 &= \frac{\tan 4^{\circ}-19'-10'' \sin 23^{\circ}-27'-27\frac{1}{2}''}{\cos 254^{\circ}-41'-10''} + \tan 254^{\circ}-41'-10'' \times \\
 &\hspace{25em} \cos 23^{\circ}-27\frac{1}{2}'' \\
 &= \frac{.0755294 \times 3980821}{-2641068} + 3.6519066 \times 9173499 \\
 &= -1138437 + 33500757 = 32362320 \\
 &[\text{For } \cos 254^{\circ}-41'-10'' = \cos (180 + 74^{\circ}-41'-10'') \\
 &\quad = -\cos 74^{\circ}-41'-10'' = -2641068 \quad \text{and} \\
 &\quad \tan 254^{\circ}-41'-10'' = \tan (180 + 74^{\circ}-41'-10'') \\
 &\quad = +\tan 74^{\circ}-41'-10'' = +3.6519066]
 \end{aligned}$$

$$\begin{aligned}
 &\therefore \tan (90^{\circ} + \text{Longitude of 11th house cusp}) \\
 &\quad = 32362320 = \tan 72^{\circ}-49'-44''
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Longitude of 11th House cusp} &= 72^{\circ}-49'-44'' \text{ minus } 90^{\circ}-0'-0'' \\
 &\quad \text{or } \underline{\underline{342^{\circ}-49'-44''}}
 \end{aligned}$$

## LONGITUDE OF CUSP OF 12TH HOUSE.

The oblique ascension of the 12th house cusp is 60° plus the R. A of the 10th house.

R A M C	plus	814°—41'—10'
		60°—0'—0'
<hr/>		

∴ Oblique ascension of 12th house is	374°—41'—10'
∴ R A of the 12th house	= 284°—41'—10'
and polar elevation is	= 8°—35'—9''

$$\begin{aligned}
 &\therefore -\cot (\text{Longitude of 12th house}) \\
 &= \frac{\tan 8^{\circ}-35'-9'' \sin 23^{\circ}-27'-27\frac{1}{2}''}{\cos 284^{\circ}-41'-10''} + \tan 284^{\circ}-41'-10'' \\
 &\hspace{25em} \times \cos 23^{\circ}-27\frac{1}{2}''
 \end{aligned}$$

$$\begin{aligned}
 \cos 284^{\circ}-41'-10'' &= \cos (360^{\circ}-75^{\circ}-18'-50'') \\
 &= \cos 75^{\circ}-18'-50''
 \end{aligned}$$

$$\begin{aligned}
&= 2535285 \\
\tan 284^{\circ}-41'-10'' &= \tan (360^{\circ}-75^{\circ}-18'-50'') \\
&= -3.8155086 \\
\therefore -\cot (\text{Longitude of 12th house}) \\
&= \frac{1509830 \times 3980821}{2535235} + 9173498 \times -3.8155086 \\
&= 2370723 - 35001552 = -32630829 \\
\therefore \cot (\text{longitude of 12th house}) &= 32630829 \\
\tan (90^{\circ}-\text{longitude of 12th house}) &= \tan 73^{\circ}-57'-42'' \\
\therefore \text{longitude of 12th house} &= 90^{\circ}-73^{\circ}-57'-42'' \\
&= 17^{\circ}-2'-18''
\end{aligned}$$

## LONGITUDE OF CUSP OF 3RD HOUSE.

As before the oblique ascension of the 3rd house will be 150 plus the R A of 10th house and therefore the R A of the 3rd house will be  $90^{\circ}$  less than its oblique ascension

$$\begin{aligned}
\therefore \text{R A of the third house} &= (314^{\circ}-41'-10'') + 156 - 90^{\circ} \\
&= 14^{\circ}-41'-10''
\end{aligned}$$

The polar elevation of the 3rd house is the same as that of the 11th house and hence equal to  $4^{\circ}-19'-10''$

$$\begin{aligned}
&\therefore -\cot (\text{Longitude of 3rd house}) \\
&= \frac{\tan 4^{\circ}-19'-10'' \sin 23^{\circ}-27'5''}{\cos 14^{\circ}-41'-10''} + \tan 14^{\circ}-11'-10'' \times \\
&\hspace{15em} \cos 23^{\circ}-27'5''
\end{aligned}$$

$$\cos 14^{\circ}-41'-10' = .9673292$$

$$\tan 14^{\circ}-41'-10' = 2620860$$

$$\therefore -\cot (\text{Longitude of 3rd house})$$

$$= \frac{.0755294 \times 3980821}{9673292} + 2620860 \times .9173198$$

$$= .03109044 + 24012453 = .27151497$$

$$\tan (90^{\circ} + \text{longitude of 3rd house}) = .27151497$$

$$= \tan (180^{\circ} + 15^{\circ}-11'-26'')$$

$$= \tan (195^{\circ}-11'-26'')$$

$$\therefore \text{Longitude of 3rd house} = 105^{\circ}-11'-26''$$

## LONGITUDE OF 2ND HOUSE.

The oblique ascension of this house is  $120^{\circ}$  more than the R A M.C and hence the R A of this house is only  $30^{\circ}$  more than the R A M.C  
 i.e. the R A of the 2nd house is  $344^{\circ}-41'-10''$

The polar elevation of this house is  $8-35'-7''$   
 $- \cot$  (Longitude of 2nd house)

$$= \frac{\tan 8^{\circ}-35'-7'' \sin 23^{\circ}-27\frac{1}{2}'}{\cos 34^{\circ}-41'-10''} + \tan 34^{\circ}-41'-10'' \times$$

$$\cos 23^{\circ}-27\frac{1}{2}'$$

$$\cos 34^{\circ}-41'-10'' = \cos (360^{\circ} - 15^{\circ}-18'-50'')$$

$$= \cos 15^{\circ}-18'-50''$$

$$= 9644984$$

$$\tan 34^{\circ}-41'-10'' = \tan (360^{\circ} - 15^{\circ}-18'-50'')$$

$$= -\tan 15^{\circ}-18'-50''$$

$$= -2738296$$

$- \cot$  (Longitude of 2nd house)

$$= \frac{1509880 \times 3980821}{9644984} + .9173498 \times -2738296$$

$$= 06231622 - 25119752 = -18888130$$

$$\therefore \cot (\text{longitude of 2nd house}) = 18888130$$

$$(i.e.) \tan (90^{\circ} - \text{longitude of 2nd house}) = \tan 10^{\circ}-41'-46''$$

$$\therefore \text{longitude of 2nd house} = 90^{\circ} - 10^{\circ}-41'-46''$$

$$= 79^{\circ}-18'-14''$$

It should be noted that in finding out the angle whose tangent or cotangent is the simplified result of the R. H. S. of the equation for finding out the longitude of the houses the respective quadrant will have to be decided with reference to the oblique ascension of the particular house so that the oblique ascension and the longitude arrived at are not very widely apart

Summing up the various results we have the cusps of the houses as follows —

	10th house	11th house	12th house	Ascdt	2nd house	3rd house
deg	312	342	17	51	79	105
mts	18	49	2	21	18	11
secs	9	44	18	44	14	26

	4th house	5th house	6th house	Descdt	8th house	9th house
deg	132	162	197	231	259	285
mts	13	49	2	21	18	11
secs	9	44	18	44	14	26

The cusps of the 10th to 3rd houses being known the cusps of 4th to 9th houses are got by adding 6 signs to the corresponding ones of the former. These are tropical longitudes and hence the amount of precession has to be subtracted to make them Nirayana and to compare with the longitudes obtained by the Hindu method.

### NIRAYANA LONGITUDES AS GOT BY TRIGONOMETRICAL METHOD.

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
deg	0	1	2	3	4	5	6	7	8	9	10	11
mts	28	26	22	19	20	24	28	26	22	19	20	24
secs	44	41	34	36	12	25	44	41	34	36	12	25
	56	26	38	21	56	30	56	46	38	21	56	30

From a comparison of the two results there may be found a difference which is due to a difference in the primary conception of a house. Even among the European astronomers there is a diversity of opinion whether to take an house based on the trisection of primevertical or of equator or of each degree of ecliptic and so on. As such nobody is competent authority to criticise the other and the best will be to cling to that which gives proper results in the experiences of each individual as the effects of houses fall really under the regions of Astrology.

The method given here is only the method in common general use and known as "Semi-arc" method. The poles of the houses are calculated for  $\frac{1}{2}$ ,  $\frac{2}{3}$  and full of the Ascensional Difference (चरफल). But it appears and appeals to the author that better results will be arrived at if instead of  $\frac{1}{2}$ ,  $\frac{2}{3}$  and full of the Ascensional difference, the प्रथमचरखंड, द्वितीयचरखंड including

The first and the full of the three charakandas a  
polar elevations of the houses

8-35-7

These have been explained at length in 344°-41-10" X  
I hope that the comparative methods of the Hindu and  
have been made clear and capable of being followed  
patient readers

cos 23°-27½

## Chapter XI

### MOON.

Periodic time of Moon or time of one sidereal }  
revolution of Moon about the earth is 27 32166 days  
No of days since epoch=41101 82153

To find out the number of revolutions and the fraction of the  
revolution left over we have to divide the latter—the number of days from  
epoch—by the periodic time The number of revolutions may be left off and  
the fraction of the revolution is converted to signs degrees etc

In the present case number of revolutions

$$= \frac{41101\ 82153}{27\ 32166} = 1504\ 367653$$

The fractional portion 367653 when converted to signs etc gives  
4s-12°-21-18 adding to this the epoch position viz 10s-21°-40-36" we  
get 3s-4°-1-54

A table of mean motion of moon is herewith appended and it can be  
used and the long division avoided

	s	dgs	mts	secs
Motion in 40000 days	0	14	21	35
do 1000 days	7	6	21	32
do 100 days	7	27	38	9
do 1 day	0	13	10	35
do 82153 of a day	0	10	49	29
Motion in 41101 82153 days is	4	12	21	20
Position at epoch	10	21	40	36
Mean longitude at Birth time	3	4	1	56

# 4th house 5th h **MEAN MOTION OF MOON.**

—dic time = 27 32166 days

deg mits secs	132			Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds	
	Degs	Minute	Seconds											
	0	13	10	35	300	11	22	54	28	50000	0	17	56	59
Th	0	26	21	10	400	7	20	32	37	60000	0	21	32	22
9th house	1	9	31	45	500	3	18	10	46	70000	0	25	7	46
These . 4	1	22	42	20	600	11	15	48	55	80000	0	28	43	10
subtrac	5	2	5	52	700	7	13	27	4	90000	1	2	18	33
obtain	6	2	19	3	800	3	11	5	14	100000	1	5	53	57
	7	3	2	14	900	11	8	43	23	200000	2	11	47	55
	8	3	15	24	1000	7	6	21	32	300000	3	17	41	53
	9	3	28	35	2000	2	12	43	5	400000	4	23	35	51
10	4	11	45	49	3000	9	19	4	37	500000	5	29	29	49
20	8	23	31	38	4000	4	25	26	9	600000	7	5	23	46
30	1	5	17	27	5000	0	1	47	42	700000	8	11	17	44
40	5	17	3	16	6000	7	8	9	14	800000	9	17	11	40
50	9	28	49	5	7000	2	14	30	47	900000	10	23	5	36
60	2	10	34	54	8000	9	20	52	19	1000000	11	28	59	31
70	6	22	20	42	9000	4	27	13	51					
80	11	4	6	31	10000	0	3	35	24					
90	3	15	52	20	20000	0	7	10	47					
100	7	27	38	9	30000	0	10	46	11					
200	3	25	16	18	40000	0	14	21	35					

## **HINDU METHOD TO FIND OUT MEAN MOON.**

Multiply the number of days from epoch by 3 and divide by 82 The quotient will be the number of revolutions With the remainder get signs, degrees minutes and seconds

Again divide the number of days by 3 The quotient will be minutes and the balance is reduced to seconds Once again divide the number of days by 260 and get minutes and seconds. The sum of all the three results will be the mean Moon The empirical correction will be 457 -36" for every 1,000 000 days additive

### **EXAMPLE.**

$$\frac{41101 \times 3}{82} = 1503 \frac{57}{82} \text{ revolutions}$$



I	Reducing $\frac{57}{82}$ of a revolution		8—10—14—38
II	$\frac{41101}{3} = 13700' - 20$	=	7—18 20—20 ✓
III	$\frac{41101}{260} = 158 - 5$	=	0—2—38—5
	Motion in 82153 of a day	=	0—10—49—29
			<hr/>
	Adding up		4—12—2—32
	Empirical correction = $\frac{457 \text{ } ^{\circ} 36'' \times 41101}{1\,000\,000}$	=	0—0—18—49
			<hr/>
			4—12—21 21
	Mean position at epoch		10—21—40—36
			<hr/>
	Mean longitude of Moon at the instant of birth is		3—4 —1—57
			<hr/>

It would be found that there is very little difference between the results arrived at previously

The next step is to find out the position of Moon's apse by all the methods

### APSE'S POSITION

Periodic time of apse	=	3232 54051 days
Number of days from epoch	=	41101 82153
Number of revolutions $\approx \frac{41101\,82153}{3232\,54051}$	=	12 715021

Reducing the decimal to signs etc we have  $8^{\circ}-17^{\circ}-24'-27''$  to which if the epoch position of moon's apse  $0^{\circ}-24^{\circ}-25'-16''$  be added we get  $9^{\circ}-11^{\circ}-49'-43''$  as the position of apse at birth

### FROM TABLES

Motion in 40000 days	$4^{\circ}-14^{\circ}-42'--2''$
do 1000 days	3—21—22—3
do 100 days	0—11—8—12
do 1 day	0—0—6—41
do 82153 day	0—0—5—29
	<hr/>
Adding motion in 41101 82153 days	8—17—24—27
	<hr/>

Position of Apse at epoch is  $0^{\circ} 24' 25'' 16''$   
 $\therefore$  Position of Moon's apse at the instant of birth is  $9^{\circ} 11' 49'' 13''$

## TABLE OF MOTION OF MOON'S APSE LINE (चंद्रोच्च).

Periodic time 3232 54051 days

Days	°	Degree	Minutes	Seconds	Days	°	Degree	Minutes	Seconds	Days	°	Degree	Minutes	Seconds
1	0	0	6	11	200	0	22	16	25	80000	3	11	1	31
2	0	0	13	22	300	1	3	24	37	10000	4	14	42	2
3	0	0	20	3	400	1	11	32	49	50000	5	18	22	32
4	0	0	26	14	500	1	25	41	2	60000	6	22	3	2
5	0	0	33	25	600	2	6	49	14	70000	7	25	43	33
6	0	0	40	6	700	2	17	57	26	80000	8	29	24	3
7	0	0	46	46	800	2	29	5	38	90000	10	3	4	34
8	0	0	53	27	900	3	10	13	51	100000	11	6	45	4
9	0	1	0	8	1000	3	21	22	3	200000	10	13	30	9
10	0	1	6	49	2000	7	12	44	6	300000	9	20	15	13
20	0	2	13	38	3000	11	4	6	9	400000	8	27	0	17
30	0	3	20	28	4000	2	25	28	12	500000	8	3	45	22
40	0	4	27	17	5000	6	16	50	15	600000	7	10	30	26
50	0	5	34	6	6000	10	8	12	18	700000	6	17	15	31
60	0	6	40	55	7000	1	29	34	21	800000	5	24	0	34
70	0	7	47	45	8000	5	20	56	24	900000	5	0	45	39
80	0	8	54	34	9000	9	12	18	27	1000000	4	7	30	43
90	0	10	1	23	10000	1	3	40	30					
100	0	11	8	12	20000	2	7	21	1					

## HINDU METHOD.

Divide the number of days from epoch by 9. The quotient gives degrees and the balance if any, is converted to minutes and seconds. Again divide the number of days by 66 and get minutes etc. Once again divide the number of days by 360 when the quotient will be seconds. The sum of the three will be the mean motion of Apse of Moon.

Empirical correction is additive at 186-15 for every 1 000 000 days

## EXAMPLE.

$$\frac{41101}{9} = 4566\frac{7}{9} \text{ degrees} = 4566^{\circ} - 46 - 40''$$

$$\frac{41101}{66} = 622\frac{49}{66} \text{ minutes} = 10^{\circ} - 22 - 45''$$

$$\frac{11101}{960} = 114\frac{51}{960} \text{ sec} = 0^{\circ} - 1 - 54$$

$$\text{Motion in 82153 of a day} = 0^{\circ} - 5 - 29$$

$$\text{Motion in 41101 82153 days} = 4577^{\circ} - 16 - 48$$

$$= 8^{\circ} - 17^{\circ} - 16 - 48$$

$$\text{Empirical correction} = \frac{41101 \times 186 - 5''}{1\,000\,000} = 0^{\circ} - 0^{\circ} - 7 - 39$$

$$\text{Position at Epoch} = 0^{\circ} - 24^{\circ} - 25 - 16$$

$$\text{Longitude of Apse of Moon at the instant of birth} = 9^{\circ} - 11^{\circ} - 49 - 43$$

All the three methods are found to give the same mean longitude of the Apse and any method can be employed

Having thus arrived at the mean positions of the Moon and her Apse we can derive the true position of hers in her own orbit

The motion of the Moon is very complicate For she has a motion of her own about herself secondly about the earth in about  $27\frac{1}{4}$  days and thirdly about the sun in the annual elliptic motion of the earth about whom she has her orbit

The earth's motion in the elliptical orbit about the sun is slowly progressing and the Moon also will have to accompany the earth about which she moves in a similar though smaller elliptical orbit

In her own orbit even the Moon's motion is not uniform as the shape and direction of the elliptical orbit is changing every moment due to the progressive motion of the Moon's apse line

All these tend to deflect or accelerate or retard the moon's motion in her own orbit and corrections due to these influences are quite necessary in finding out the correct mean position from which only the true position can be computed by the laws of elliptic orbits

The corrections are —(1) Annual variation (2) Evection (3) Variation (4) Equation of centre and lastly (5) Reduction These are explained as follows

# EXAMPLE.

$$\frac{41101}{9} = 4566\frac{7}{9} \text{ degrees} = 4566^{\circ}-46'-10''$$

$$\frac{41101}{9} = 4566\frac{7}{9} \text{ degrees} = 10^{\circ}-22'-15''$$

corrections will give Moon's true p = 0^{\circ}-1'-51''  
 necessary to refer her position to the ec = 0^{\circ}-5'-29''  
 the plane of the ecliptic at an angle of 5^{\circ}-7''  
 sed hereby will be on the same lines as that of 1577^{\circ}-16'-18''  
 of cliptic is to the equator The formula for the 27^{\circ}-16'-45''  
 may be used with the necessary changes With such 7'-39''  
 minary examination we have to proceed with the meth 16''  
 apply it in the present example

Mean Moon at instant of birth		3^{\circ}-1^{\circ}-1
Mean Apse do		9-11-19-4
Mean Sun do		2-29-42-40

Now take the net result of the various corrections due to the position of the observer, (चरप्राणकलांतर देशांतरबाहुफलं) on page 72 and multiply it by the daily velocity of Moon Apse and Sun separately, divide the product by 21600 and the respective results applied to them will give the mean longitudes corrected to the place of birth The net result is 28'-7 धन(+)

$$\text{Correction for Moon is } \frac{790'-35'' \times 28'-7''}{21600'} = 1'-2'' (+)$$

$$\text{do Apse is } \frac{6'-41'' \times 28'-7''}{21600} = 0-1'' (+)$$

$$\text{do Sun is } \frac{6'-41'' \times 59'-8''}{21600'} = 0'-4'' (+)$$

Applying these to the respective mean longitudes we get their rectified values

Corrected Moon's mean longitude		8^{\circ}-4^{\circ}-2^{\circ}-58'
do Apse's		9-11-49-14
do Sun's		2-29-42-44

With these corrected ones only we have to find out Annual variation, Evection Variation and Equation of centre of the Moon

# EXAMPLE.

$$\frac{41101}{9} = 4566^{\circ} \text{ degrees} = 4566^{\circ} - 46 - 40$$

$$41101 = 622^{\text{fe}} \text{ minutes} = 10^{\circ} - 22 - 45''$$

This is  $8580'' \sin \phi \sin \left( \frac{\quad}{\quad} \right)$   $= 0^{\circ} - 1 - 54$

where  $\phi = \text{Mean Moon} - \text{Mean sun}$

In the present case Moon is  $3^{\circ} - 21 - 23''$   $0^{\circ} - 5 - 29$

sun is  $2^{\circ} - 23 - 29$   $577^{\circ} - 16 - 48$

$$\phi = 0^{\circ} - 4^{\circ} - 27'' - 16 - 48$$

$$= 4^{\circ} - 20 - 14 \quad 7 - 39$$

$$\text{Variation} = -8580'' \sin (4^{\circ} - 20 - 14) \times 16$$

$$\sin \left( \frac{4 - 20 - 14 + 88^{\circ} - 22 - 31}{2} \right)$$

$$\sin \left( \frac{4^{\circ} - 20 - 14'' - 88^{\circ} - 22 - 31}{2} \right)$$

$$= -8580'' \sin (4^{\circ} - 20 - 14) \times \sin (46^{\circ} - 21 - 23'') \times \sin (-42 - 1 - 8'')$$

$$= 8580 \sin (1^{\circ} - 20 - 14'') \sin (46^{\circ} - 21 - 23'') \sin (43^{\circ} - 1 - 8'')$$

$$= 8580 \times 075646 \times 723653 \times 66937 = 314'' = 5 - 14$$

( $\sin -42 - 1 - 8$  is  $-\sin 42 - 1 - 8$  which minus sign renders the minus sign before 8580 plus)

## TABLE OF VARIATION

Argument — (Moon — Sun) If more than 180 its defect from 360 will be the argument but the sign of variation correction will be the opposite of that given in the table

Deg	Var at on	Effect on Velocity	Deg	Var at on	Effect on Velocity	Deg	Var at on	Effect on Velocity
0	10 0	+ 14 24	6	7 9	13 9	12	14 9	11 54
1	1 12	14 11	7	8 20	12 56	13	15 15	11 29
2	2 24	13 59	8	9 31	12 44	14	16 20	11 5
3	3 36	13 46	9	10 42	12 31	15	17 24	10 40
4	4 47	13 34	10	11 52	12 19	16	18 26	10 16
5	5 58	13 21	11	13 1	12 6	17	19 26	9 51

117	-33	47	+9	9	159	-21	36	+13	21	171	-11	18	+15	30	
118	33	14		9	32	160	23	11	13	36	172	10	1	15	28
119	32	39		9	55	161	22	14	18	51	173	8	45	15	26
150	32	0		10	18	162	21	15	14	6	174	7	29	15	24
151	31	16		10	41	163	20	13	14	21	175	6	14	15	22
152	30	31		11	4	164	19	10	14	36	176	4	68	15	20
153	29	45		11	27	165	18	31	14	51	177	3	13	15	18
154	28	57		11	50	166	17	27	15	6	178	2	29	15	16
155	28	8		12	13	167	16	17	15	21	179	-1	14	15	14
156	27	17		12	36	168	15	6	15	36	180	-0	0	+15	12
157	26	25		12	51	169	13	51	15	34		+0			
158	-25	31	+13	6		170	-12	35	+15	32					

Note—The effect on velocity due to this correction retains the sign given here and does not depend for its sign on the argument whether within 180 or more than 180

In the example taken variation function is  $4^{\circ} 20' - 14''$  This is written in 180 and as such this itself will be the argument to enter the table

Variation for  $4^{\circ}$  of argument is  $+4' - 47''$  effect on vel  $+13 - 34''$

do for  $5'$  of argument is  $+5' - 58''$ , do  $+13 - 21''$

Variation for  $4^{\circ} - 26' - 14''$  is  $+4 - 47''$  plus  $0 - 24''$  or  $+5 - 11''$

So also effect on velocity is  $+13 - 30''$

Having thus got the annual variation variation and evection we have to apply their net result to the mean Moon before finding out the equation of centre correction for Moon

Annual variation	=	$1 - 20'' +$
Evection	=	$20 - 49'' +$
Variation	=	$5 - 14'' +$
Total	=	$27 - 23 +$

Mean moon at birth rectified is  $3^{\circ} - 4^{\circ} - 2' - 55''$  applying to the total  $27 - 23$  to this we get  $3^{\circ} - 4^{\circ} - 30' - 21''$  This is the moon which has to be used for finding out the equation of centre

Position of Moon	=	$3^{\circ} - 4^{\circ} - 30' - 21''$
Position of Apse	=	$9^{\circ} - 11^{\circ} - 49' - 41''$

Mean anomaly or (or Apse—Moon)=  $6^{\circ} - 7^{\circ} - 19' - 23''$

Equation of centre  $= 5562 \sin nt [4 - 2745 \cos nt] + 37 \sin 3nt$   
 here  $nt = 187^\circ - 19' - 23''$

$$\therefore \sin nt = \sin 187^\circ - 19' - 23'' = \sin (180^\circ + 7^\circ - 19' - 23'')$$

$$= -\sin 7^\circ - 19' - 23'' = -.1274781$$

$$\cos nt = \cos 187^\circ - 19' - 23'' = \cos (180^\circ + 7^\circ - 19' - 23'')$$

$$= -\cos 7^\circ - 19' - 23'' = -.9918414$$

$$\therefore \text{Eqn of centre in seconds of arc} = 5562 \times -.1274781 \times$$

$$[4 - 2745 \times -.9918414]$$

$$+ 37 \sin 3(187^\circ - 19' - 23'')$$

$$= -3098 + 37 \sin (561^\circ - 58' - 9'')$$

$$= -3098 + 37 \sin (360^\circ + 201^\circ - 58' - 9'')$$

$$= -3098 + 37 \sin (201^\circ - 58' - 9'')$$

$$= -3098 - 37 \times .351 = -3098 - 13 = -3111''$$

$$= -(51' - 51'')$$

The equation being negative, has to be subtracted from the Moon  $3^\circ - 4' - 30'' - 21''$  when we get her true position in her own orbit. It will be therefore  $3^\circ - 4' - 30'' - 21''$  minus  $51' - 51''$  or  $3^\circ - 8' - 38'' - 30''$ .

## EFFECT ON VELOCITY.

Moon's velocity at the instant, is

$$= n \left\{ 1 - 2e \cos nt + 5e^2 \cos^2 nt - \frac{5e^2}{2} \right\}$$

where  $n = 790'.6$  and  $e = .0549$

It reduces to

$$784.63 - \cos nt (86.8 - 11.914 \cos nt)$$

In the present example,  $\cos nt = -.9918414$

$$\therefore \text{Velocity in mts of arc} = 784.63 + .9918 (86.8 + 11.914 \times .9918)$$

$$= 784.63 + 97.795 = 882.425$$

$$\text{Effect of Evection and Variation on Velocity} = +15.867$$

$$+ 13.5$$

---


$$901.792$$


---

$$\therefore \text{Moon's Velocity in orbit} = \underline{911' - 48''}$$

# TABLE OF EQUATION OF CENTRE OF MOON.

Argument—(Apse—Rectified Moon) If argument is within  $180^\circ$ , refer table directly If more than  $180^\circ$  the same subtracted from  $360^\circ$  will be the argument but the equation of centre will be negative then

Deg	Equation of Centre			Eqn of Velocity	Deg	Equation of Centre			Eqn of Velocity
	°	'	"			°	'	"	
0	0	0	0	-80 12	34	3 19 39	-72 50	68 5 40 40	-48 40
1	0	6	10	80 15	35	3 24 55	72 22	69 5 48 24	42 33
2	0	12	19	80 18	36	3 30 6	71 54	70 5 46 0	41 26
3	0	18	28	80 22	37	3 35 15	71 12	71 5 48 31	40 19
4	0	24	37	80 25	38	3 40 21	70 30	72 5 50 57	39 12
5	0	30	46	80 28	39	3 45 24	69 48	73 5 53 17	37 54
6	0	36	54	80 31	40	3 50 28	69 6	74 5 55 30	36 3
7	0	43	2	80 28	41	3 55 17	68 24	75 5 57 38	35 15
8	0	49	9	80 25	42	4 0 9	67 42	76 5 59 40	34 2
9	0	55	17	80 22	43	4 4 57	67 0	77 6 1 34	32 44
10	1	1	24	80 18	44	4 9 42	66 18	78 6 3 29	31 27
11	1	7	29	80 15	45	4 14 21	65 26	79 6 5 5	30 9
12	1	13	33	80 12	46	4 18 58	64 54	80 6 6 42	28 52
13	1	19	35	79 58	47	4 23 29	64 12	81 6 8 11	27 34
14	1	25	35	79 45	48	4 27 57	63 30	82 6 9 36	26 17
15	1	31	38	79 31	49	4 32 21	62 35	83 6 10 52	24 59
16	1	37	30	79 18	50	4 36 41	61 41	84 6 12 3	23 42
17	1	43	25	79 4	51	4 40 58	60 46	85 6 13 7	22 16
18	1	49	18	78 51	52	4 45 11	59 52	86 6 14 1	20 51
19	1	55	10	78 37	52	4 49 20	58 57	87 6 14 55	19 25
20	2	0	59	78 24	54	4 53 24	58 3	88 6 15 37	18 0
21	2	6	45	78 10	55	4 57 28	57 8	89 6 16 16	16 34
22	2	12	31	77 57	56	5 1 16	56 14	90 6 16 52	15 9
23	2	18	16	77 43	57	5 5 3	55 19	91 6 17 10	13 43
24	2	24	0	77 30	58	5 8 41	54 25	92 6 17 25	12 18
25	2	29	44	77 2	59	5 12 10	53 30	93 6 17 37	10 52
26	2	35	28	76 34	60	5 15 31	52 36	94 6 17 10	9 27
27	2	41	10	76 6	61	5 18 52	51 29	95 6 17 37	8 1
28	2	46	52	75 38	62	5 22 10	50 22	96 6 17 25	6 36
29	2	52	33	75 10	63	5 25 25	49 15	96 6 17 8	5 6
30	2	58	4	74 42	64	5 28 37	48 8	98 6 16 44	3 36
31	3	3	34	74 14	65	5 31 46	47 1	99 6 16 11	2 6
32	3	9	0	73 46	66	5 34 52	45 54	100 6 15 32	0 36
33	3	14	19	-73 18	67	5 37 52	-44 17	101 6 14 17	+ 0 54



102	6 18 53	+2 24	129	5 6 17	+48 11	156	2 44 6	+76 48
103	6 12 54	3 54	130	5 2 11	44 39	157	2 36 41	77 34
104	6 11 48	5 24	131	4 57 57	46 8	158	2 31 11	78 20
105	6 10 34	6 54	132	4 53 41	47 36	159	2 24 35	79 6
106	6 9 12	8 24	133	4 49 17	48 56	160	2 17 55	79 52
107	6 7 45	9 54	134	4 44 48	50 15	161	2 11 16	80 38
108	6 6 9	11 24	135	4 40 13	51 35	162	2 4 36	81 24
109	6 4 27	12 57	136	4 35 32	52 54	163	1 57 54	82 10
110	6 2 36	14 29	137	4 30 45	54 14	164	1 51 11	82 56
111	6 0 42	16 2	138	4 25 53	55 33	165	1 44 25	83 42
112	5 58 38	17 34	139	4 20 53	56 53	166	1 37 39	84 28
113	5 56 28	19 7	140	4 15 52	58 12	167	1 30 50	85 14
114	5 54 12	20 39	141	4 10 42	59 32	168	1 24 1	86 0
115	5 51 43	22 12	142	4 5 27	60 51	169	1 17 10	86 29
116	5 49 17	23 44	143	4 0 7	62 11	170	1 10 17	86 57
117	5 46 40	25 17	144	3 54 42	63 30	171	1 3 19	87 26
118	5 44 05	26 49	145	3 49 13	64 37	172	0 56 19	87 54
119	5 41 4	28 22	146	3 43 37	65 43	173	0 49 19	88 23
120	5 38 6	29 54	147	3 37 57	66 50	174	0 42 19	88 51
121	5 35 3	31 23	148	3 32 14	67 56	175	0 35 17	89 20
122	5 31 51	32 51	149	3 26 24	69 3	176	0 28 15	89 48
123	5 28 33	34 20	150	3 20 34	70 9	177	0 21 12	90 17
124	5 25 8	35 48	151	3 14 43	71 16	178	0 14 9	90 45
125	5 21 36	37 17	152	3 8 47	72 22	179	0 7 5	91 14
126	5 17 54	38 45	153	3 2 15	73 29	180	0 0 0	+91 42
127	5 14 10	40 14	154	2 56 39	74 35			
128	5 10 18	+41 42	155	2 50 21	+75 42			

Let us now obtain the equation of centre and equation of velocity from the tables, in the present instance

$$\begin{aligned} \text{Mean anomaly of Moon} &= 6^{\circ}-7'-19'-23'' \\ \text{or} &187^{\circ}-19'-23'' \end{aligned}$$

As this is more than 6 signs on  $187^{\circ}$ , subtracting this from  $360^{\circ}$ , we get  $172^{\circ}-40'-37''$  as the argument for referring to the tables.

For  $172^{\circ}$ , equation of centre is  $0^{\circ}-56'-19''$  and

For  $173^{\circ}$  do is  $0^{\circ}-49'-19''$

$\therefore$  For  $172^{\circ}-40'-37''$  it is  $51'-35''$ , this is negative as the mean anomaly is greater than  $180^{\circ}$

$\therefore$  Equation of centre is  $(- )51'-35''$ , correction as per next table with the mean anomaly is  $(- )14'$   $\therefore$  Corrected equation of centre is  $(- )51'-49''$ . Equation of velocity read from the tables is  $88'-14''(+)$

As the equation of centre is negative, subtracting it from the mean moon rectified with Evection Variation and annual variation, we get the

$$\begin{array}{r} 3^s - 4^o - 30' - 21'' \\ \text{minus } 51' - 49'' \\ \hline 3^s - 3^o - 38' - 32'' \end{array}$$

longitude of the Moon as

## CORRECTIONS TO EQUATION OF CENTRE OF MOON.

POSITIVE					"	NEGATIVE						
0 60	120	180	240	300	0	60	120	180	240	300	360	
1 59	121	179	241	299	2	61	119	181	239	301	359	
2 58	122	178	242	298	4	62	118	182	238	302	358	
3 57	123	177	243	297	6	63	117	183	237	303	357	
4 56	124	176	244	296	8	64	116	184	236	304	356	
5 55	125	175	245	295	10	65	115	185	235	305	355	
6 54	126	174	246	294	11	66	114	186	234	306	354	
7 53	127	173	247	293	13	67	113	187	233	307	353	
8 52	128	172	248	292	15	68	112	188	232	308	352	
9 51	129	171	249	291	17	69	111	189	231	309	351	
10 50	130	170	250	290	19	70	110	190	230	310	350	
11 49	131	169	251	289	20	71	109	191	229	311	349	
12 48	132	168	252	288	22	72	108	192	228	312	348	
13 47	133	167	253	287	23	73	107	193	227	313	347	
14 46	134	166	254	286	24	74	106	194	226	314	346	
15 45	135	165	255	285	26	75	105	195	225	315	345	
16 44	136	164	256	284	27	76	104	196	224	316	344	
17 43	137	163	257	283	29	77	103	197	223	317	343	
18 42	138	162	258	282	30	78	102	198	222	318	342	
19 41	139	161	259	281	31	79	101	199	221	319	341	
20 40	140	160	260	280	32	80	100	200	220	320	340	
21 39	141	159	261	279	34	81	99	201	219	321	339	
22 38	142	158	262	278	34	82	98	202	218	322	338	
23 37	143	157	263	277	35	83	97	203	217	323	337	
24 36	144	156	264	276	35	84	96	204	216	324	336	
25 35	145	155	265	275	35	85	95	205	215	325	335	
26 34	146	154	266	274	36	86	94	206	214	326	334	
27 33	147	153	267	273	36	87	93	207	213	327	333	
28 32	148	152	268	272	37	88	92	208	212	328	332	
29 31	149	151	269	271	37	89	91	209	211	329	331	
30 30	150	150	270	270	37	90	90	210	210	330	330	

As for the equation of velocity the following table should be used when the argument is more than 180

MOON'S EQUATION OF VELOCITY FROM  
180° TO 360° OF MEAN ANOMALY.

180	+91	42	217	+76	55	254	+28	27	291	-26	8	328	-65	48
181	91	43	218	75	50	255	26	55	292	27	26	329	66	30
182	91	44	219	74	45	256	25	24	293	28	45	330	67	12
183	91	45	220	73	40	257	24	52	294	30	3	331	67	54
184	91	46	221	72	35	258	23	21	295	31	22	332	68	36
185	91	47	222	71	30	259	21	49	296	32	40	333	69	18
186	91	48	223	70	25	260	20	18	297	33	59	334	70	0
187	91	49	224	69	20	261	18	46	298	35	17	335	70	42
188	91	50	225	68	15	262	17	15	299	36	36	336	71	24
189	91	51	226	67	10	263	15	43	300	37	54	337	71	54
190	91	52	227	66	5	264	13	12	301	39	2	338	72	24
191	91	53	228	65	0	265	11	41	302	40	9	339	72	54
192	91	54	229	63	40	266	10	10	303	41	17	340	73	24
193	91	29	230	62	21	267	8	39	304	42	24	341	73	54
194	91	5	231	60	1	268	7	8	305	43	32	342	74	24
195	90	40	232	59	42	269	5	37	306	44	39	343	74	54
196	90	16	233	58	22	270	4	0	307	45	47	344	75	24
197	89	51	234	57	3	271	2	35	308	46	54	345	75	54
198	89	27	235	55	43	272	+1	4	309	48	2	346	76	24
199	89	2	236	54	24	273	-0	27	310	49	9	347	76	54
200	88	38	237	53	4	274	1	58	311	50	17	348	77	24
201	88	12	238	51	45	275	3	29	312	51	24	349	77	38
202	87	49	239	50	25	276	5	0	313	52	22	350	77	52
203	87	24	240	49	6	277	6	26	314	53	20	351	78	6
204	87	0	241	47	38	278	7	52	315	54	18	352	78	20
205	86	15	242	46	10	279	9	18	316	55	16	353	78	34
206	85	30	243	44	42	280	10	44	317	56	14	354	78	48
207	84	45	244	43	14	281	12	10	318	57	12	355	78	2
208	84	0	245	41	46	282	13	36	319	58	10	356	79	16
209	83	15	246	40	18	283	15	2	320	59	8	357	79	30
210	82	30	247	38	50	284	16	28	321	60	6	358	79	44
211	81	45	248	37	22	285	17	54	322	61	4	359	79	58
212	81	0	249	35	54	286	19	20	323	62	2	360	-80	12
213	80	15	250	34	26	287	20	46	324	63	0			
214	79	30	251	32	58	288	22	12	325	63	42			
215	78	45	252	31	30	289	23	31	326	64	24			
216	+78	0	253	+29	58	290	-24	49	327	-65	6			

In the present instance the mean anomaly is  $187^{\circ}-19'-23''$  The equation of velocity is found from the table as  $91-49+$  The velocity found out at end of page 104 will apply had the mean anomaly been  $172^{\circ}-40'-37''$ .

$$\begin{array}{lcl}
 \therefore \text{Correct velocity of Moon} & = & 790-35''+ \\
 = \text{Mean velocity} + & & 91-49'' \\
 + \text{equation of velocity due to evection} & & 29-22' \\
 + \text{equation of velocity due to variation} & & \text{-----} \\
 + \text{equation of velocity due to eccentricity} & & 911'-46'' \\
 & & \text{-----}
 \end{array}$$

Having got the true position of the Moon in her orbit we have to find her longitude on the ecliptic as the longitudes are distances measured along the ecliptic. The process resembles that in finding out the R A from a given longitude. The planes of transformation therein are those of the ecliptic and the equator the angle between them having a mean value of  $23^{\circ}-27\frac{1}{2}'$ . While in the case of the moon her orbit is inclined to the ecliptic at an angle of about  $5^{\circ}-9'$  or more nearly  $5^{\circ}-8'$  and as such the relation between the distances of the moon from a point common to the ecliptic and her orbit measured respectively along the ecliptic and her orbit, and the inclination of the Moon's orbit to the ecliptic has to be established.

Suppose M be the position of the Moon in her own orbit at a particular instant P the foot of the vertical through M and N the common point on the orbit and the ecliptic.



The common point is called a *Node* and there are in general two nodes for each orbit by virtue of its intersecting the ecliptic while projected on the celestial sphere. These Nodes apply for all planetary orbits and they are different for different orbits. That Node through which the planet traverses from the south to the north is called the *Ascending Node* and its opposite point will be the *Descending Node*.

As a rule these Nodes always recede. Nodes are known in Hindu Astronomy as “(पातानि)” and the Moon's nodes are called *Rahu* (राहु) and *Ketu* (केतु).

# TABLE OF MOTION OF NODES OF MOON.

Periodic time = 6793 39477 days

Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds
1	0	0	3	11	300	0	15	53	52	50000	4	9	37	57
2	0	0	6	22	400	0	21	11	49	60000	9	29	33	32
3	0	0	9	32	500	0	26	29	47	70000	3	19	29	8
4	0	0	12	43	600	1	1	47	44	80000	9	9	24	43
5	0	0	15	54	700	1	7	54	41	90000	2	29	20	19
6	0	0	19	5	800	1	12	23	39	100000	8	19	15	54
7	0	0	22	16	900	1	17	41	36	200000	5	8	31	49
8	0	0	25	26	1000	1	22	59	33	300000	1	27	47	43
9	0	0	28	37	2000	3	15	59	7	400000	10	17	8	38
10	0	0	31	48	3000	5	8	58	40	500000	7	6	19	32
20	0	1	3	35	4000	7	1	58	14	600000	3	25	35	26
30	0	1	35	23	5000	8	24	57	47	700000	0	14	51	21
40	0	2	7	11	6000	10	17	57	20	800000	9	4	7	15
50	0	2	38	59	7000	0	10	56	54	900000	5	23	23	10
60	0	3	10	46	8000	2	3	56	27	1000000	2	12	89	4
70	0	3	42	34	9000	3	26	56	1					
80	0	4	14	22	10000	5	19	55	35					
90	0	4	46	9	20000	11	9	51	11					
100	0	5	17	57	30000	4	29	46	46					
200	0	10	35	55	40000	10	19	42	22					

## HINDU METHOD.

Divide the number of days past from epoch by 566 The quotient will be signs Reduce the balance to degrees etc Again divide the number of days by 3281 The quotient will be minutes and the balance is reduced to seconds The difference between the two will be the motions of Rahu The empirical cor-rection is 348-10 for every 1000000 days to be subtracted from the Hindu positions to get its modern value

## ACCORDING TO HINDU METHOD

$$\frac{41101}{566} = 72^{\circ} \frac{349}{566} = 0^{\circ}-18^{\circ}-29'-54''$$

$$\frac{41101}{3281} = 12' \frac{1729}{3281} = 0^{\circ}-0^{\circ}-12'-32''$$

$$\text{Difference} = 0^{\circ}-18^{\circ}-17'-22''$$

Empirical correction (negative)

$$= \frac{348' - 10'' \times 41101}{1000000} = 0^s - 0^m - 14' - 19''$$

$$\text{Net} = 0 - 18 - 3 - 3$$

$$\text{Motion in 82153 of a day} = 0 - 0 - 2 - 37$$

$$0 - 18 - 5 - 40$$

racting this as before from epoch we get

$$0^s - 12^m - 12' - 38'' \text{ minus}$$

$$0^s - 16^m - 5' - 40''$$

n Rahu

$$11^s - 24^m - 6' - 53''$$

## TRANSFERENCE OF MOON'S LONGITUDE ALONG THE ORBIT TO THE ECLIPTIC.

(REDUCTION)

Let M be the position of (say) moon along her own orbit and let P the corresponding foot of the vertical through M the pole of the ecliptic. If  $i$  be the angle of inclination of the plane of the lunar orbit to the ecliptic we can at once find a relation between NP, NM and  $i$ .



For, since MP is a vertical through the pole of the ecliptic  $\Delta MNP$  is a spherical one and as such Napier's parts could be applied.

Sine of middle part = product of tangents of adjacent parts. If  $i$  is the middle part, NM and NP are the adjacent parts and their Napier's parts are  $(90^\circ - i)$  and NP.

$$\therefore \sin (90^\circ - i) = \tan (90^\circ - NM) \tan NP$$

$$(i.e.) \cos i = \cos NM \tan NP$$

$$\therefore \tan MP = \cos i \tan NM \text{ whence NP is obtained}$$

Further as MP is the height above the ecliptic measured along the vertical through M and the pole of the ecliptic MP is the latitude.

If MP is treated as the middle part NM and  $i$  are its opposites and their Napier's parts are MP,  $(90^\circ - NM)$  and  $(90^\circ - i)$ .

$$\therefore \text{ sine of middle part} = \text{ product of cosines of opposites}$$

$$(i.e.) \sin MP = \cos (90^\circ - NM) \cos (90^\circ - i)$$

$$(i.e.) \sin MP = \sin NM \sin i$$

This gives the latitude in general but the lunar latitude is subject to perturbations which affect the latitudes to a considerable extent. If correct latitude of the Moon is to be found the following correction in addition to that obtained by the formula  $\sin M P = \sin i \sin N M$  will have to be made

It is  $+522 \sin (3 M - 2 S - N)$  where M S and N stand respectively for Moon Sun and Rahu

## TO FIND REDUCTION IN THE EXAMPLE.

Longitude of Moon as arrived at previously is	$3^{\circ} - 1' - 38'' - 32''$
Distance of node (राहु)	$11^{\circ} - 24' - 6'' - 56''$
Nodal distance (पातोनचदकद्र)	$= 3^{\circ} - 9' - 31'' - 36''$
or	$99^{\circ} - 31' - 36''$

As this is more than  $90^{\circ}$  it is nearer to the descending Node than the ascending one. The distance to the descending node is  $180^{\circ}$  minus  $99^{\circ} - 31' - 36''$  or  $80^{\circ} - 28' - 24''$ . Now

$$\begin{aligned}
 \tan NP &= \cos i \times \tan NM \\
 &= \cos 5^{\circ} - 8' - 8'' \times \tan 80^{\circ} - 28' - 24'' \\
 &= 9959683 \times 5.9587307 \\
 &= 5.9347067 \\
 NP &= 80^{\circ} - 26' - 8''
 \end{aligned}$$

As NM was originally got by subtracting from 180 the Nodal distance along the orbit the distance from Node to the foot of the vertical through the Moon and the pole the ecliptic is got by subtracting this from 180

$NM = 180^{\circ} - 80^{\circ} - 26' - 8'' = 99^{\circ} - 33' - 52''$  This is the Nodal distance along the ecliptic to which if longitude of Node is added we get the longitude of the Moon as measured along the ecliptic

Nodal distance	$= 3^{\circ} - 9' - 33'' - 52''$
Longitude of Node	$= 11^{\circ} - 24' - 6'' - 56''$
Longitude of Moon	$= 3^{\circ} - 3' - 40'' - 48''$

As it is thus difficult to find out the distance along the ecliptic the following table may be used with some advantage. The procedure is this Subtract the Longitude of portion of Node from that of Moon in her own orbit

Empirical correction (negative)

$$= \frac{348' - 10'' \times 41101}{1000000} = 0^s - 0^m - 14' - 19''$$

$$\text{Net} \quad \quad \quad = 0 - 18 - 3 - 3$$

$$\text{Motion in 82153 of a day} = 0 - 0 - 2 - 37$$

$$\underline{\quad \quad \quad} \\ 0 - 18 - 5 - 40$$

Subtracting this as before from epoch we get  $0^s - 12^m - 12' - 33''$  minus

$$\underline{\quad \quad \quad} \\ 0^s - 18^m - 5' - 40''$$

Mean Rahu

$$\underline{\quad \quad \quad} \\ \underline{\quad \quad \quad} \\ 11^s - 24^m - 6' - 53''$$

## TRANSFERENCE OF MOON'S LONGITUDE ALONG THE ORBIT TO THE ECLIPTIC. (REDUCTION)

Let M be the position of (say) Moon along her own orbit and let P the corresponding foot of the vertical through M and the pole of the ecliptic. If  $i$  be the angle of inclination of the plane of the lunar orbit to the ecliptic we can at once find out a relation between NP, NM and  $i$ .



For since MP is a vertical through the pole of the ecliptic  $\triangle MNP$  is a right spherical one and as such Napier's parts could be applied.

Sine of middle part = product of tangents of adjacents. If  $i$  is the middle part NM and NP are the adjacents and their Napier's parts are  $(90^\circ - i)$ ,  $(90^\circ - NM)$  and NP.

$$\therefore \sin (90^\circ - i) = \tan (90^\circ - NM) \tan NP$$

$$(i.e.) \cos i = \cos NM \tan NP$$

$$\therefore \tan MP = \cos i \tan NM \text{ whence NP is obtained}$$

Further as MP is the height above the ecliptic measured along the vertical through M and the pole of the ecliptic MP is the latitude.

If MP is treated as the middle part NM and  $i$  are its opposites and their Napier's parts are MP,  $(90^\circ - NM)$  and  $(90^\circ - i)$ .

$$\therefore \text{ sine of middle part} = \text{ product of cosines of opposites}$$

$$(i.e.) \sin MP = \cos (90^\circ - NM) \cos (90^\circ - i)$$

$$(i.e.) \sin MP = \sin NM \sin i$$



For  $80^\circ$  we get reduction as  $2'-24''$ , for  $81^\circ$  it is  $2'-6''$

$\therefore$  For  $80^\circ-28'-24''$  reduction is  $2'-17''$ .

The Nodal distance being in the II quadrant, the correction due to this effect is positive

Position of Moon in her own orbit is  $3^\circ-3'-38'-32''$ .

Reduction correction is plus  $2'-17''$ .

$\therefore$  Longitude of Moon referred to ecliptic is  $3^\circ-3'-40'-49''$ .

This is the same as that got by the trigonometrical method This is called (पात or राहुसंस्कार)

This final answer of the longitude of Moon is quite sufficient but there are some slight optional corrections which may be of some interest for those who would find the time and pleasure to indulge in

The following functions are necessary to find out the correction —

- 1) Moon—Sun (अर्कोनचंद्र) (see under variation)
- 2) Sun's mean anomaly as defined by us (रविमंदकेन्द्र)
- 3) Moon's mean anomaly (चंद्रमंदकेन्द्र)
- 4) Sun—Moon's Apse (इंदूच्चोनरविकेन्द्र) (see under Evection, which

function is  $A+M-2S=(\overline{M}-\overline{S}-\overline{S}-\overline{A})$

$$=(\text{अर्कोनचंद्रकेन्द्र}-\text{इंदूच्चोनरविकेन्द्र})$$

- 5) Nodal distance of Moon (पातोनचंद्रकेन्द्र)

- 6) Nodal distance of Sun (पातोनरविकेन्द्र) (ie) distance of Sun from Moon's Node,

These functions have been all derived incidental to other working except the last one which is not very difficult to find out Having found items (1) to (6), and assigning them the letters  $a$  to  $f$  for them we have the optional corrections as

$$\begin{aligned} & -155 \sin (2a+b)+198 \sin (a+b-d) \\ & +112 \sin (b-c)+73 \sin (b+c)+85 \sin (c+2c)-81 \sin 2f, \end{aligned}$$

in seconds of arc

## IN THE EXAMPLE TAKEN.

Sun	Moon	Sun's Apse	Moon's Apse	Node
2	3	2	9	11
29	4	18	11	24
42	30	47	49	6
44	21	38	44	56

## FROM THESE.

$a = \text{Moon-Sun}$	$= 4^{\circ}-20'$	$2a+b = 358^{\circ}$
$b = \text{Sun's Apse-Sun}$	$= 349^{\circ}-5'$	$a+b-d = 186^{\circ}$
$c = \text{Moon's Apse-Moon}$	$= 187^{\circ}-19'$	$b-c = 162^{\circ}$
$d = \text{Sun-Moon's Apse}$	$= 167^{\circ}-53'$	$b+c = 176^{\circ}$
$e = \text{Moon-Node}$	$= 100^{\circ}-23'$	$c+2e = 28^{\circ}$
$f = \text{Sun-Node}$	$= 95^{\circ}-36'$	$2f = 191^{\circ}$

$\therefore$  Correction in seconds of arc

$$\begin{aligned}
 &= -155 \sin 358^{\circ} + 198 \sin 186^{\circ} + 112 \sin 162^{\circ} + 73 \sin 176^{\circ} \\
 &\quad + 85 \sin 28^{\circ} - 81 \sin 191^{\circ} \\
 &= 155 \sin 2^{\circ} - 198 \sin 6^{\circ} + 112 \sin 18^{\circ} + 73 \sin 4^{\circ} + 85 \sin 28^{\circ} \\
 &\quad + 81 \sin 11^{\circ} \\
 &= 155 \times 035 - 198 \times 105 + 112 \times 309 + 73 \times 07 + 85 \times 47 \\
 &\quad + 81 \times 191 \\
 &= 545 - 2079 + 3461 + 511 + 3995 + 1547 \\
 &= +10059 - 2079 = +7980 = +(1' - 20'')
 \end{aligned}$$

Adding this, since positive, to the true longitude of Moon already found out we get correct longitude of Moon as  $3^{\circ}-3^{\circ}-40'-48''$  plus  $1'-20''$  or  $3^{\circ}-3^{\circ}-42'-8''$

## IMPROVED HINDU METHOD OF DETERMINING THE TRUE LONGITUDE OF MOON.

The Moon's mean longitude is  $3^{\circ}-4^{\circ}-2'-58''$  and its apse is  $9^{\circ}-11^{\circ}-49'-44''$ . The mean anomaly is got as usual by subtracting the Moon from the Apse It is  $9^{\circ}-11^{\circ}-49'-44''$  minus  $3^{\circ}-4^{\circ}-2'-58''$  or  $6^{\circ}-7^{\circ}-46'-46''$ . Then the Hindu table of equation of centre is entered into. The rules for finding out the argument for entering into the tables are the same as those given in the chapter under sun

The mean anomaly in this case is in the III quadrant Hence the argument is got by subtracting  $180^{\circ}$  from the mean anomaly It is  $7^{\circ}-46'-46''$ .

for 7th item, equation of centre is	$36'-40''$
and equation of velocity is	$68'-0''$
for 8th item equation of centre is	$41'-52''$
and equation of velocity is	$67'-50''$
$\therefore$ Equation of centre for $7^{\circ}-46'-46''$ is	$40'-43''$ (—)
and equation of velocity is	$67'-52''$ (+)

[The former is (—) and the latter (+) as the mean anomaly is in the III quadrant—vide instructions under sun's chapter]

# HINDU TABLES OF MOON'S EQUATION OF CENTRE.

Deg	Equation of Centre	Eqn of Velocity	Deg	Equation of Centre	Eqn of Velocity	Deg	Equation of Centre	Eqn of Velocity	
0	0	0 69 35	32	159 27 57 51		64	270 38 29 32		
1	5 15 68 84		33	163 53 57 10		65	272 54 29 27		
2	10 30 68 32		34	168 15 56 34		66	275 6 27 21		
3	15 45 68 28		35	172 36 55 50		67	277 11 26 15		
4	20 59 68 23		36	176 52 55 8		68	279 12 25 8		
5	26 13 68 16		37	181 6 54 25		69	281 8 24 1		
6	31 27 68 9		38	185 16 53 44		70	282 59 22 54		
7	36 40 68 0		39	189 23 52 36		71	284 45 21 46		
8	41 52 67 50		40	193 27 52 9		72	286 25 20 38		
9	47 5 67 39		41	197 27 51 22		73	288 0 19 29		
10	52 14 67 25		42	201 24 50 34		74	289 30 18 20		
11	57 25 67 13		43	205 17 49 46		75	290 54 17 10		
12	62 33 66 58		44	209 5 48 55		76	292 13 16 1		
13	67 40 66 42		45	212 50 48 5		77	293 27 14 51		
14	72 46 66 24		46	216 32 47 20		78	294 36 13 40		
15	77 51 66 6		47	220 9 46 20		79	295 39 12 30		
16	82 54 65 46		48	223 42 45 27		80	296 36 11 19		
17	87 57 65 25		49	227 11 44 33		81	297 28 10 8		
18	92 57 65 3		50	230 36 43 38		82	298 15 8 57		
19	97 57 64 39		51	233 57 42 42		83	298 56 7 46		
20	102 54 64 15		52	237 13 41 45		84	299 31 6 34		
21	107 49 63 49		53	240 26 40 48		85	300 1 5 23		
22	112 43 63 22		54	243 33 39 50		86	300 27 4 11		
23	117 34 62 54		55	246 37 38 51		87	300 16 2 59		
24	122 23 62 29		56	249 36 37 51		88	301 0 1 48		
25	127 9 61 58		57	252 30 36 51		89	301 8 0 36		
26	131 54 61 23		58	255 20 35 50		90	301 11 0 0		
27	136 36 60 50		59	258 5 34 49					
28	141 15 60 17		60	260 45 33 47					
29	145 52 59 42		61	263 20 32 44					
30	150 27 59 6		62	265 52 31 50					
31	154 58 58 29		63	268 17 30 36					

∴ Longitude of Moon

$$= \left\{ \begin{array}{r} 3^{\circ}-4^{\circ}-2'-58'' \text{ minus} \\ \quad \quad \quad 40-43 \\ \hline 3^{\circ}-3^{\circ}-22'-15'' \end{array} \right.$$

$$\begin{array}{r} \text{velocity} = 790' - 85'' + \\ \quad \quad \quad 67' - 52'' \quad (\text{equation of velocity}) \\ \hline 858' - 27'' \end{array}$$

These are subject to further corrections as follows —

The correction for bringing the longitude of Moon as found out with the help of the Hindu tables to close approximation to the modern accurate longitude of the Moon has been given by a later astronomer after the founders of the eighteen siddhantas

The true longitudes of Sun Moon with their daily velocities and the mean longitude of Apse are all noted down Subtract the mean longitude of Apse from the sun's position, the result will be the *Indoochona Ravi Kendram* (इंदूचोनरविकेंद्रं) Find the sine argument first, and also the cosine argument by subtracting the former from  $90^{\circ}$  Find the equations of centre for the two arguments either of which can never exceed  $90^{\circ}$ , being highest item in the table

Then subtract sun's longitude from that of Moon This is called *Arkona chandra kendram* (अर्कोनचंद्रकेन्द्रं). As before find out the equation of centre for the sine argument and also that for the cosine argument

Arrange the results thus —

### A (इंदूचोनरविकेंद्रं) INDOOCHONA RAVI KENDRAM.

Eqn of centre of sine argument (सुत्रज्या)... . x

Eqn of centre of cosine do (कोटिज्या) .... y

### B (अर्कोनचंद्रकेन्द्रं) ARKONA CHANDRA KENDRAM.

Eqn of centre of sine argument (सुत्रज्या) .. .. x

Eqn of centre of cosine do (कोटिज्या).. .. y

Find  $\frac{y \times 10}{2474}$  where 2474 is a constant The quotient will be minutes and remainder is converted to seconds. The quotient is positive if A and B are both within  $180^{\circ}$  or both greater than  $180^{\circ}$ , and negative other-

wise. That is, positive if both are मेपादि or both तुलादि and negative otherwise.

The result got is added to or subtracted from as found out above, to the mean velocity, when we got the true velocity of Moon.

Next find out  $\frac{z \times y}{527}$  in minutes. Multiply the result by the ratio of true velocity arrived at above to the mean velocity  $790' - 35''$ . The final result will be the correction to be applied to the Moon's longitude already arrived at, as per the rules laid hereunder.

I When B is within  $180^\circ$

Correction is positive if A is between  $90^\circ$  and  $270^\circ$  and negative if otherwise

II When B is greater than  $180^\circ$

Correction is positive, if A is between  $270$  and  $90$  and is negative if otherwise

Expressed in Sanskrit

दृक्संस्कारः शुद्धपक्षे, धनं यदि इंदूच्चोत्तरविकेंद्रं कक्ष्यादि, ऋणं यदि मकरादि ।

कृष्णपक्षे एतयोः विपरीतं । तच्च कक्ष्यादिऋणं, मकरादिधनं ।

This method gives very satisfactory results but when function B approaches  $180^\circ$  or  $360^\circ$ : for, the correction of this method involves a sine function which becomes zero ultimately. Apparently there will be no correction at all. This cannot be; for even though variation may become zero, being function B, Evection which is  $(B-A)$  need not be zero.

The following suggestion may be used. The eqn of centre corresponding to the sine argument of A as not been used at all. Multiply it by 60 and divide by 527. find the quotient. As before the quotient is multiplied by the ratio of the true and mean velocities. This will be positive or negative according as A is within  $180^\circ$  or greater than  $180^\circ$ .

## EXAMPLE.

(True Longitude of Sun)	(True Longitude of Moon)	(Mean Longitude of Apse)	(Sun— Apse)	(Moon— Sun)
2	3	9	5	0
29	3	11	17	4
21	22	49	31	0
22	15	44	38	53
	<hr/>		<hr/>	<hr/>
	858		A	B
	27			

Sine	Agt of A is 0—12—28—22	Equation of centre is	64'—58"
Cosine	Agt of A is 2—17—31—38	do	is 294'—3"
Sine	Agt of B is 0—4—0—53	do	is 21'—4"
Cosine	Agt of B is 2—25—9—7	do	is 300'—27"

$$\frac{y \times v}{2474} = \frac{300' - 27'' \times 294' - 3''}{2474} = \frac{88348}{2474} = 35' - 42''$$

Now A and B are both within  $180^\circ$  whence this  $35-42$  is positive and therefore additive to the velocity of the moon arrived at viz.  $858'-27''$ , we get  $894'-9''$

$$\text{Next find out } \frac{z \times y}{527} = \frac{21' - 4'' \times 294' - 3''}{527} \times \frac{894' - 9''}{858' - 27''} = 13' - 17''$$

This is the correction to be applied to the longitude of Moon already found out B is within  $180^\circ$  (i.e.) शुक्रपक्ष and A is between  $90^\circ$  and  $270^\circ$  or वसुधोदरे. Hence the correction is positive Adding this to the true longitude of Moon already found out, we get  $3^\circ - 3' - 23' - 15''$  plus  $13' - 17'' = 3^\circ - 3' - 35' - 32''$  Sine argument of A is  $64' - 55''$ . The correction due to this is

$$\frac{64' - 55'' \times 60}{527} \times \frac{894' - 9''}{790' - 35''} = 8' - 22''$$

A is within  $180^\circ$  hence this is positive. Adding this also, to the previously corrected moon we get

$$\begin{array}{r} 3^\circ - 3' - 35' - 32'' \\ \text{plus} \qquad \qquad \qquad 8' - 22'' \\ \hline 3^\circ - 3' - 43' - 54'' \end{array}$$

$\therefore$  The correct longitude of Moon is  $3^\circ - 3' - 43' - 54''$  and velocity  $884'-9''$  This reaches a closer approximation to the longitude of Moon found out by the modern method but anyhow, the daily velocity fails to reach the standard of accuracy of the more refined way The higher excentricity of the Moon of the Modern astronomers is responsible for the higher value of the velocity

A further improvement to the Hindu method is herewith given —

The mean longitudes of the Sun Moon and Apse are first noted down. The functions (Moon—Sun) and (Sun—Apse),  $\alpha$  and  $\beta$  respectively are found out If they are determined, the correction to be applied to the mean Moon can be got by one formula viz, in minutes of arc.

$\frac{1}{11} [-11.9 \text{ Equation of centre for } (a-\beta)]$

$-4 \text{ Equation of centre for } a + 7.1 \text{ Equation of centre for } 2a]$

The correction thus got is applied to the mean Moon. This corrected Moon is subtracted from the position of Apse when we get the mean anomaly. The Hindu table of equation of centre is entered into and the equation of centre as also the effect on velocity are noted down. These are subject to further corrections as follows.

## MULTIPLIER FOR EQUATION OF CENTRE.

This is  $\frac{1887 (4 \sin^{15} \cos nt)}{6021}$  The equation of centre previously

found out should be multiplied with this multiplier and applied to the mean longitude of Moon when the correct true longitude of Moon is got.

## MULTIPLIER FOR EQUATION OF VELOCITY.

It is  $\frac{5.97 + \cos nt (86.8 - 11.914 \cos t)}{68.6 \cos nt}$

The equation of velocity got from the tables is multiplied by this multiplier and then applied to the mean daily velocity of Moon viz 790-35. A further correction to the velocity is as follows.

$$\frac{1}{3012} \left[ \begin{aligned} &-14.9 \times \text{equation of centre of } \cos \arg \text{ of } (a-\beta) \\ &-4 \times \text{equation of centre of } \cos \arg \text{ of } a \\ &+15.2 \times \text{equation of centre of } \cos \arg \text{ of } 2a \end{aligned} \right]$$

If this is also got with due regard to the sign of the cosine function and applied to the Moon's daily velocity the result will be the true daily velocity.

## EXAMPLE.

$$a = \text{Mean Moon} - \text{Mean Sun} = \begin{array}{r} \phantom{0}8 \phantom{0}0 \phantom{0}0 \phantom{0}0 \\ 3-4-2-59- \\ 2-29-42-14 \\ \hline 0-4-20-14 \end{array}$$

$$\beta = \text{Mean Sun} - \text{apse} = \begin{array}{r} \phantom{0}2 \phantom{0}29 \phantom{0}42 \phantom{0}44 \\ 9-11-49-14 \\ \hline 5-17-53-0 \end{array}$$

## Correction to mean Moon

It is  $\frac{1}{60} \left[ \begin{array}{l} -149 \text{ equation of centre } (a-\beta) \\ -4 \text{ equation of centre } a \\ +71 \text{ equation of centre } 2a \end{array} \right]$

$a-\beta = 6^s-16'-27'' = 196''-27''$ , in this case the sine argument is  $16^s-27'$  and its equation of centre is  $-85'-11''$  [The sign of the equation of centre may not please be ignored]

$$a = 0^s-4'-20'' = 4'-20''.$$

The sine argument is  $4'-20''$  and its equation of centre is  $22'-24''$ .

$$2a = 0^s-8'-40'' = 8'-40''$$

The sine argument is  $8'-40''$  and its equation of centre is  $45'-19''$ .

Hence the correction to be applied to moon is

$$\begin{aligned} & \frac{1}{60} [-149 \times -(85'-11'') - 4 \times (22'-24'') + 71(45'-19'')] \\ &= \frac{1}{60} [(1269-20'') - (8'-58'') + (321'-46'')] \\ &= \frac{1}{60} [(1582'-8'')] = \underline{+26'-22''} \end{aligned}$$

Corrected mean longitude of Moon is  $(3^s-4'-2''-58'') + (26'-22'')$   
 $= 3^s-4'-29'-20''$

$$\begin{aligned} \text{Mean anomaly} &= \left\{ \begin{array}{l} 9^s-11'-4''-44'' \text{ minus} \\ 3^s-4'-29'-20'' \end{array} \right. \\ & \quad \underline{6^s-7'-20'-24''} \\ \text{or} \quad & 167^{\circ}-20'-24'' \end{aligned}$$

The argument for referring to the Hindu tables is  $7^{\circ}-20'-24''$  and the effect of the equation of centre will be negative as the mean anomaly is greater than  $180$

Equation of centre for  $7^{\circ}$  is  $36'-40''$  and for  $8^{\circ}$  is  $11'-52''$

Equation of velocity for  $7^{\circ}$  is  $68'-0''$  and for  $8^{\circ}$  is  $67'-50''$

Equation of centre for  $7^{\circ}-20'-24''$  is  $38'-26'' (-)$

Equation of velocity for  $7^{\circ}-20'-24''$  is  $67'-56'' (+)$

Multiplier for equation of centre  $\frac{1887}{6024} (4-2745 \cos nt)$

$$= \frac{1887}{6024} (4-2745 \cos 187^{\circ}-20'-24'')$$

$$= \frac{1887}{6024} (4-2745 \times -99182)$$

$$= \frac{1887}{6024} \times 427275 = 1339$$

$$\begin{aligned} \therefore \text{Correct equation of centre} &= -(38'-26'') \times 1339 \\ &= -(51'-4'') \end{aligned}$$



$$\therefore \text{True longitude of Moon} \left\{ \begin{array}{r} 3-4-29-30 \\ \quad \quad \quad 51-45 \\ \hline 3-3-37-35 \\ \quad \quad \quad + \quad 1-20 \\ \hline 3-3-38-55 \end{array} \right.$$

$$\text{बाहुफलं} = \frac{1}{16} \times (+21' - 22'')$$

(Annual variation =  $\frac{1}{16}$  of Sun's  
equation of centre reversed)

The last may be also added to the mean moon and the mean anomaly found before finding out the equation of centre

Multiplier's for equation of velocity

$$\begin{aligned} & \frac{597 + \cos nt (868 - 11914 \cos nt)}{686 \cos nt} \\ &= \frac{597 - 9918(868 - 11914 \times -9918)}{686 \times -9918} \\ &= \frac{597 - 9918(868 + 1182)}{-6803} = \frac{597 - 97811}{-6803} \\ &= \frac{-91541}{-6803} = 135 \end{aligned}$$

$$\begin{aligned} \text{Rectified equation of velocity} &= 135 \times 67' - 56'' \\ &= 91' - 43'' \end{aligned}$$

Further correction for velocity —

$$\begin{aligned} & \frac{1}{301.2} \left[ \begin{array}{l} -14.9 \times \text{equation of centre of } \cos \arg \text{ of } (a-\beta) \\ -4 \times \text{equation of centre of } \cos \arg \text{ of } a \\ + 15.2 \times \text{equation of centre of } \cos \arg \text{ of } 2a \end{array} \right] \end{aligned}$$

$(a-\beta) = 195^\circ - 27'$ . The cosine of this is negative. The equation of the centre of the cosine arg of this is the same as that of its sine argument viz  $73^\circ - 33'$ .

$a = 4^\circ - 20'$ . The cosine is positive and the equation of centre of the cosine argument of this is the same as that of its sine argument viz  $87^\circ - 40'$

$2a = 8^\circ - 40'$ . The cosine is positive and the equation of centre of the cosine argument of this is the same as that of its sine argument viz  $81^\circ - 20'$ .

Now equation of centre for  $73^\circ - 33'$  is  $288' - 50''$

do for  $87^\circ - 40'$  is  $300' - 18''$

do for  $81^\circ - 20'$  is  $297' - 44''$

$$= \left[ \frac{-14.7 \times -288.8 - 4 \times 300.3 + 15.2 \times 297.7}{301.2} \right]$$

### Correction to mean Moon

It is  $\frac{1}{60} \left[ \begin{array}{l} -14.9 \text{ equation of centre } (a-\beta) \\ -4 \text{ equation of centre } a \\ +7.1 \text{ equation of centre } 2a \end{array} \right]$

$a-\beta = 6^s-16^m-27^s = 196^m-27^s$ , in this case; the sine argument is  $16^m-27^s$  and its equation of centre is  $-85'-11''$ . [The sign of the equation of centre may not please be ignored]

$$a = 0^s-4^m-20^s = 4^m-20^s.$$

The sine argument is  $4^m-20^s$  and its equation of centre is  $22'-24''$ .

$$2a = 0^s-6^m-40^s = 8^m-40^s$$

The sine argument is  $8^m-40^s$  and its equation of centre is  $45'-19''$ .

Hence the correction to be applied to moon is

$$\begin{aligned} \frac{1}{60} & [-14.9 \times -(85'-11'') - 4 \times (22'-24'') + 7.1(45'-19'')] \\ & = \frac{1}{60} [+(1269'-20'') - (8'-38'') + (321' - 46'')] \\ & = \frac{1}{60} [(1582'-8'')] = \underline{+26'-22''} \end{aligned}$$

Corrected mean longitude of Moon is  $(3^s-4^m-2'-58'') + (26'-22'')$   
 $= 3^s-4^m-29'-20''$

$$\begin{aligned} \text{Mean anomaly} &= \left\{ \begin{array}{l} 9^s-11^m-49'-44'' \text{ minus} \\ 3^s-4^m-29'-20'' \\ \hline 6^s-7^m-20'-24'' \\ \text{or} \quad 187^m-20'-24'' \end{array} \right. \end{aligned}$$

The argument for referring to the Hindu tables is  $7^m-20'-24''$  and the effect of the equation of centre will be negative as the mean anomaly is greater than  $180^\circ$ .

Equation of centre for  $7^m$  is  $36'-40''$  and for  $8^m$  is  $41'-52''$

Equation of velocity for  $7^m$  is  $68'-0''$  and for  $8^m$  is  $67'-50''$

$\therefore$  Equation of centre for  $7^m-20'-24''$  is  $38'-26'' (-)$

Equation of velocity for  $7^m-20'-24''$  is  $67'-56'' (+)$

Multiplier for equation of centre  $\frac{1.887}{6024} (4-2745 \cos nt)$

$$= \frac{1.887}{6024} (4-2745 \cos 187^m-20'-24'')$$

$$= \frac{1.887}{6024} (4-2745 \times -.99182)$$

$$= \frac{1.887}{6024} \times 1.27275 = 1.339$$

$$\begin{aligned} \therefore \text{Correct equation of centre} &= -(38'-26'') \times 1.339 \\ &= -(51'-41'') \end{aligned}$$

∴ True longitude of Moon

$$\text{बाहुफलं} = 1^{\circ} \times (+21' - 22')$$

(Annual variation  $-1^{\circ}$  of Sun's equation of centre reversed)

$$\left\{ \begin{array}{r} 87 \\ = 3-4-29-30 \\ - 51-45 \\ \hline 3-3-37-35 \\ + 1-20 \\ \hline 3-3-38-55 \end{array} \right.$$

The last may be also added to the mean moon and the mean anomaly found before finding out the equation of centre.

Multiplier's for equation of velocity

$$\begin{aligned} & \frac{5.97 + \cos nt (86.8 - 11.914 \cos nt)}{68.6 \cos nt} \\ &= \frac{5.97 - 9918(86.8 - 11.914 \times -9918)}{68.6 \times -9918} \\ &= \frac{5.97 - 9918(86.8 + 11.82)}{-68.03} = \frac{5.97 - 97.811}{-68.03} \\ &= \frac{-91.841}{-68.03} = 1.35 \end{aligned}$$

$$\begin{aligned} \text{Rectified equation of velocity} &= 1.35 \times 67' - 50'' \\ &= 91' - 43'' \end{aligned}$$

Further correction for velocity —

$$\begin{aligned} & \frac{1}{801.2} \left[ -14.9 \times \text{equation of centre of } \cos \arg \text{ of } (a - \beta) \right. \\ & \quad \left. - 4 \times \text{equation of centre of } \cos \arg \text{ of } a \right. \\ & \quad \left. + 15.2 \times \text{equation of centre of } \cos \arg \text{ of } 2a \right] \end{aligned}$$

$(a - \beta) = 196^{\circ} - 27'$ . The cosine of this is negative. The equation of the centre of the cosine arg of this is the same as that of its sine argument viz  $73^{\circ} - 33'$ .

$a = 4^{\circ} - 20'$ . The cosine is positive and the equation of centre of the cosine argument of this is the same as that of its sine argument viz  $85^{\circ} - 40'$ .  $2a = 8^{\circ} - 40'$ . The cosine is positive and the equation of centre of the cosine argument of this is the same as that of its sine argument viz  $81^{\circ} - 20'$ .

Now equation of centre for  $73^{\circ} - 33'$  is  $288' - 50''$

do for  $85^{\circ} - 40'$  is  $300' - 18''$

do for  $81^{\circ} - 20'$  is  $297' - 44''$

$$= \left[ \frac{-14.7 \times -288.8 - 4 \times 300.3 + 15.2 \times 297.7}{801.2} \right]$$

$$= \frac{4245'4 - 120'12 + 4525'0'}{3012} = \frac{8650'28}{3012} = 28' - 44''$$

$$\therefore \text{True Velocity} = \begin{cases} 790' - 35'' + \\ 91' - 43'' + \\ 28' - 44'' \\ \hline 911' - 2'' \end{cases}$$

It will be found that of the two improvements to the Hindu method, the latter is better as it will approximate to the true longitude of Moon arrived at by the regular method before the 'Reduction' and subsequent correction is applied. Reduction also could be applied without much effort as there has been laid out a table for the purpose for each degree of argument of Nodal distance. The subsequent corrections are only optional.

The former method of the improvement suggested is from one of the 18 siddhantas—It is learnt to be in Lomash—and it is a pity that though the method speaks very high of and stands in good stead of the inventor does not seem to be in use nor of any remembrance even among many of the old school of almanac-compilers. It reaches in close nearness and can well replace the old plain method without any special corrections due to the various causes of Evection Variation and Annual Variation leaving off the other minor optional corrections. It is found that due to the archaic methods having ignored these causes and their consequent effects on the mean Moon the true longitude of Moon differs in some cases by 4°—18' in the maximum and by 3°—9' in the minimum. It is regretted that even in Kerala where Indian astronomy is yet fostered the leading Astronomers are deaf to suggestions in this direction and it is hoped that this book will bring about the desired reform.

Both the methods are so given as to base their needs only on the Hindu tables of equation of centre and the trigonometrical tables and it is hoped that the followers of this book will not have any difficulty in practical working.

## Chapter XII.

### PLANETS.

The planets are Mercury Venus Earth Mars Jupiter Saturn, Uranus and Neptune of which the first two are called Inferior and the rest Superior planets. These all perform their ceaseless journeys in their own elliptical orbits about the Sun in one of their foci of their respective orbits.

Their distances from the Sun increase in the order enumerated above and the angle formed between the line joining a planet to the Sun and a fixed direction of the First point of Aries—fixed if the first point of Aries were considered to have no precession—is called the *Heliocentric longitude* or longitude in its own orbit

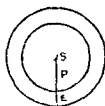
The earth unlike the sun is not fixed but has a motion of its own in an orbit which has already been defined in its elements. The relative motion of a planet to an observer on the earth will sometimes be accelerated retarded stationary or retrograde depending on their mutual positions with respect to each other and the sun

The position at which a planet will be seen by an observer at the centre of the earth, is called the *Geocentric longitude*.

Let S E and P be the sun earth and planet in their respective orbits, the sun being fixed and hence having no orbit

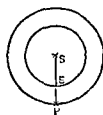
In the case of Inferior planets their orbits will always fall between the sun and the earth's orbit while those of Superior ones always outside the earth's orbit

In the adjoining figure P is an Inferior planet and hence its orbit is within that of the earth



When P intervenes S and E the phenomenon is called 'an Inferior conjunction and when S intervenes P and E it is an opposition

In the case of Superior planets their orbits always fall outside the earth's orbit



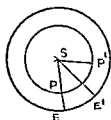
When E intervenes S and P it is a superior conjunction and as before when S intervenes P and E it is an opposition

Thus when the centres of S E and P are collinear or in one and the same straight line the phenomenon is a superior or an inferior conjunction according as E or P is the intervening body

The interval between one conjunction or one opposition of the same nature is called the *Synodic period* and that of one revolution of the planet about the sun with respect to the fixed stars or the fixed direction of  $\gamma$  is called the *Sidereal period* or *Periodic Time*

The sidereal period which is necessary to find out the mean heliocentric longitude cannot be easily found out unless with the help of the Synodic Period. Instead of taking the interval between the synodic positions at random it is usually taken at two very favourable positions like Transits of Venus which come far between each other. Thus the interval between two such transits divided by the number of synodic revolutions will give the exact synodic period of the planet.

## I. INFERIOR PLANET.



Let P an inferior planet and E the earth. S P E is the line of centres at the time of conjunction. Let after a time P be at P' and E at E'.

Then  $\angle PSE$  will be necessarily greater than  $\angle ESE$  for P has to describe a smaller orbit.

Let P be the sidereal period of planet and E that of the earth also let S be the synodic period of the planet. Now  $\angle PSE = \angle ESE + \angle ESP$ .

If the interval of time chosen be an unit it reduces to

$\frac{1}{P} = \frac{1}{E} + \frac{1}{S}$  of which S and E are known. Hence P would be found very easily.

## II. SUPERIOR PLANET.

In the above write E for P and P for E we get

$$\frac{1}{E} = \frac{1}{P} + \frac{1}{S} \text{ or } \frac{1}{P} = \frac{1}{E} - \frac{1}{S} \text{ whence P is found out}$$

Having thus known the sidereal period the advance of mean longitude of a planet for a given interval of time can be very easily found out which when added to the epoch position gives the mean longitude at the particular instant reckoned from the epoch.

The planes of the different orbits are not in the same plane as that of the ecliptic but inclined to it at different angles but very small.

Each orbit being elliptical has got its own apse line formed by joining the Aphelion and Perihelion and the Apsidal line has a forward motion with the exception of that of Venus.

The two points where the orbit of a planet when projected on the celestial sphere cuts the ecliptic are called the Nodes in the same correspondence to the Nodes of the Lunar orbit. These nodes also have a backward motion.

How the apse lines have a forward and the apse line of Venus and the Nodal lines of all planets have a backward motion belong to the investigation of Physical Astronomy which we do not propose to enter into

Thus the following elements should be known first to find out the geocentric latitude at a given place and a given instant —

- I a) Positions in own orbit at epoch (मध्यग्रह)
- b) Periodic times (प्रदक्षिणाकाल)
- II a) Mean longitude of Apse lines (मदोच्चानि)
- b) Annual velocity of Apse line (मदोच्चवर्गति)
- III a) Mean longitudes of line of Nodes (पातानि)
- b) Annual velocity of Nodal line (पातवर्गति)
- IV a) Ex-centricity of the orbit of the planets (वद्व्युत्तयः)
- b) Semi major axes (मध्यममदकर्णाः)
- V a) Inclination of plane of planet's orbit to the ecliptic (परमविशेषानि)

Having arrived at these we can next proceed to find out the geocentric latitude as follows —

First we can find out the position of the body whose geocentric longitude is to be found out in its own orbit for which we require Mean heliocentric longitude at epoch of the body and that of the apse line of its orbit the excentricity of the orbit and the mean daily motion of the planet and that of the apse line

Take the number of days elapsed from epoch and divide it by the number of days in one sidereal revolution or periodic time. The quotient will be revolutions and the balance is converted to signs degrees minutes and seconds. Add the result to the epoch mean longitude of the planet which result will give its mean position at the instant. So also for the position of the apse line. The excentricity of the orbit is known. Thus the elliptical orbit of the planet with its mean position is completely defined and its true longitude in its own orbit can be found out with the help of the same formula which we used to find the Earth's position in its orbit but taking care to use the particular value of the excentricity of the planet chosen.

Subtract the mean longitude of the planet from the mean longitude of its Apse when we get the mean anomaly. Then apply the mean anomaly

in the formula

$$\frac{e \sin nt (4 - 5e \cos nt)}{2} + \frac{e^3}{12} (13 \sin 3nt - 3 \sin nt)$$

The last term involving  $e^3$  may be neglected except in the case of planets whose  $e$  is sufficiently great as to cause any appreciable change in the result

The formula when substituted with the known quantities will give the equation of centre with the proper sign which when applied to the mean heliocentric longitude of the planet gives us the true heliocentric longitude of the planet

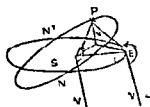
The distance of the planet from the sun is called the radius vector (मदकर्णः) This has also to be known. It is given by the formula

$$r = \frac{l}{1 - e \cos \theta} = \frac{a(1 - e^2)}{1 - e \cos \theta} \text{ where } \theta \text{ is the true anomaly as defined by us and } a \text{ the length of the semi major axis of the planetary orbit}$$

As the true heliocentric longitude and the length of the radius vector thus having been arrived at the planet is located in its own orbit

It has already been mentioned that the planes of the orbits of the planets are not in the same plane as that of the ecliptic but inclined to it

Let P be an inferior planet in its own orbit and E the earth in the earth's orbit or the ecliptic which is merely the projection on the celestial sphere of the sun's apparent path



Let PM be drawn  $\perp$ ar to the plane of the ecliptic since as aforesaid the path traced by E is only the ecliptic when projected on the celestial sphere. Let S' and E' be the direction of the First point of Aries

Now for an observer at S the planet will be seen through angle  $\angle SP$  while the same for an observer at E will be seen through an angle  $\angle EP$  in the positive or counter clockwise direction. This angle  $\angle EP$  is the geocentric longitude but since longitudes are measured along the ecliptic  $\angle EM$  will be the geocentric longitude. It may appear that  $\angle EP$  and  $\angle EM$  should be the same as M, E, P is one plane. It is true from the present figure but it will be made clear later that there will be a slight difference due to the inclination of the planetary orbit to the ecliptical plane



When P is at a Node, (i e) at either point where the two planes cut—one, the plane of the planetary orbit and the other the plane of the ecliptic,—it is in the same plane as that of the ecliptic. Hence it has no latitude. But when it is exactly  $90^\circ$  from the either Node it has the maximum latitude which is then equal to the inclination of the orbit. Aptly it has been defined by our Hindu Astronomers as परमविक्षेपं. In any case the  $\angle$  PSM gives the latitude of the planet

In  $\triangle$ SPM, SP = radius vector (known)

$\angle$  PSM = latitude (found out)

$\therefore$  SM is known, for,  $SM = SP \cos \angle$  PSM. Now in  $\triangle$ MSE MS is known, SE = radius vector of earth (known) and  $\angle$  MSE = difference of Heliocentric longitudes of P and E

$\therefore EM^2 = SM^2 + SE^2 - 2 SM \cdot SE \cos \angle$  MSE. All the quantities on the R H S being known EM is found out

We have  $\frac{SE}{\sin \angle$  SME} =  $\frac{ME}{\sin \angle$  MSE}

$\therefore \sin \angle$  SME =  $\frac{SE}{ME} \sin \angle$  MSE whence  $\angle$  SME is found out

Of the angles at E the angles MES and  $\angle$  SE $\hat{\Gamma}$ ' are known from which the angle  $\angle$   $\Gamma$ 'EM can be easily found out

$\angle$   $\Gamma$ 'EM =  $\angle$   $\Gamma$ 'SE +  $\angle$  ESM +  $\angle$  SME, for, by producing SE to any point X, the exterior angle XEM =  $\angle$  ESM +  $\angle$  SME

To each add  $\angle$   $\Gamma$ 'EX

We get  $\angle$   $\Gamma$ 'EX +  $\angle$  XEM =  $\angle$  ESM +  $\angle$  SME +  $\angle$   $\Gamma$ 'EX

(i e)  $\angle$   $\Gamma$ 'EM =  $\angle$  ESM +  $\angle$  SME +  $\angle$   $\Gamma$ 'SE

for E $\hat{\Gamma}$ ' is the direction  $\parallel$  to S $\hat{\Gamma}$ ' and hence  $\angle$   $\Gamma$ 'SE =  $\angle$   $\Gamma$ 'EX

In the case of superior planets S P is always greater than SE whence the angle SPE and consequently  $\angle$  SME will always be acute.

Whereas, in the case of inferior planets, there may seem some difficulty in taking the correct value of  $\angle$  SME, for as S P and hence SM is always less than SE angle SME should be always greater than  $\angle$  MES. The angle got as the value of  $\angle$  SME should be fixed only with reference to this. If this is found to be less than  $\angle$  MES, the supplement of the value of  $\angle$  SME originally got should be taken as the correct value to be applied to

∠TSE for finding out the geocentric longitude This is so because the angle whose sine =  $\frac{SE}{ME} \sin \angle MSE$  can be either say  $\theta$  or  $(180^\circ - \theta)$

### EXAMPLE.

In  $\triangle SME$  suppose  $\angle ESM = 331^\circ$  and suppose the value of the angle as obtained from the formula  $\sin \angle SME = \frac{SE}{ME} \sin \angle MSE$  be  $69^\circ$  Now  $(180^\circ - 69^\circ)$  or  $111^\circ$  also will have the same sine To determine which of the two has to be taken follow this procedure

Take  $\angle ESM$  as given if less than  $180$  if greater than  $180$  then take its defect from  $360^\circ$  In this case it is  $360^\circ - 331^\circ$  or  $29^\circ$   $\angle SME = 69^\circ$   $MES$  should be  $-180^\circ - (29^\circ + 69^\circ) = -82^\circ$ , but  $\angle MES$  should be always less than  $\angle SME$  which is not the case Therefore the correct value of  $\angle SME$  should be the supplement of  $69^\circ$  (i.e.)  $111^\circ$  This has to be applied to  $\angle TSM$  to get  $\angle TEM$

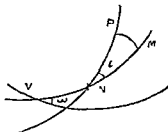
In finding out  $\angle MSE$  care should be taken for  $\angle MS^\circ$  is the Heliocentric longitude (H L) of the planet and  $\angle ES^\circ$  that of Earth while we have got only the longitude of Sun Therefore adding  $180^\circ$  to the longitude of sun we get the H L of the earth

The rest is very simple

It should be noted that M is the projection of the planet on the ecliptic Referring to the annexed figure NP is known by subtracting the longitude of Node from the H L of the planet

$\angle PNM$  is known as the inclination of the planetary orbit to the ecliptic

Thus the spherical  $\triangle NPM$  is solved and NM and PM are both found out For as the longitudes are measured along the ecliptic we want only NM and not NP



$$\cos z = \tan NM \cot NP$$

$$\tan NM = \cos z \tan NP$$

• NM is known

This is akin to the process of finding out the प्राणकलांतरसंस्कार, which

$$\text{is given as } \frac{2 \tan (\text{Nodal distance}) \times \sin^2 \frac{z}{2}}{1 + \tan^2 (\text{Nodal distance}) \cos z}$$

The same table can be used by applying the multiplier  $\frac{\sin^2 \frac{1}{2}}{\sin^2 \frac{w}{2}}$

which will serve for all practical purposes,

So also  $\sin PM = \sin i \sin NP$  whence  $PM$  the heliocentric latitude is also obtained. This is same as the angle  $PSM$  we used in the previous pages and the longitudes  $\Upsilon^M$  will measure the same angle  $\Upsilon SM$  shown in the figure page

We shall now work out each planet separately to afford detailed example to the followers and readers of this work

## Chapter XIII.

### MARS.

#### ELEMENTS

1	Mean longitude at epoch	$7^{\circ}-1^{\circ}-48'-54''$
2	Mean Hel longitude of Apse	$4^{\circ}-11'-18'-0''$
3	Mean Hel longitude of Node	$0^{\circ}-26^{\circ}-54'-18''$
4	Length of semi major axis	1.5287
5	Excentricity of orbit	0.09331
6	Inclination of orbit to ecliptic	$1^{\circ}-51' 1''$
7	Periodic time	686.980 days
8	Annual motion of Apse	$= +16''.86$
9	Annual motion of Nodes	$= -22''.74$

These though already given in the consolidated table have been reproduced here for ready reference

No of days elapsed from epoch to the moment of birth is	41101.82153
Periodic time in days is	686.980
$\therefore$ No of revolutions	$= \frac{41101.82153}{686.98}$
	$= 59.82972$

Converting the decimal portion alone to signs etc, we get

	$9^s - 28^\circ - 41' - 57''$
Position at epoch	$7^s - 1^\circ - 48' - 54''$
$\therefore$ Mean Mars at birth	$5^s - 0^\circ - 30' - 51''$

## HINDU METHOD.

Divide the number of days by 687 The quotient will be revolutions with the balance get signs degrees minutes and seconds Let this be A Again divide the number of days by 1788 The quotient will be minutes and the remainder is reduced to seconds Let this be B The sum of these two will be the mean motion of Mars (Empirical correction for 1 000 000 days is 356' additive)

$$\frac{4110182153}{687} = 59 \frac{56882153}{68700000} \text{ revolutions}$$

$$\frac{56882153}{687} \text{ of a revolution} = 9^s - 28^\circ - 4' - 21'' \text{ (A)}$$

$$\frac{4110162}{1788} = 22' - 59'' \text{ (B)}$$

$$\text{Adding up} = 9^s - 28^\circ - 27' - 20''$$

$$\text{Empirical correction for } \left. \begin{array}{l} 41101 \text{ days} \end{array} \right\} = \frac{356 \times 41101}{1000000} = 14' - 38'' \text{ (additive)}$$

$$\therefore \text{Corrected mean motion} = 9^s - 28^\circ - 41' - 58''$$

$$\text{Epoch mean Mars} = 7^s - 1^\circ - 48' - 54''$$

$$\therefore \text{Mean longitude of Mars at the } \left. \begin{array}{l} \text{moment of birth} \end{array} \right\} = 5^s - 0^\circ - 30' - 52''$$

## FROM TABLES.

	$\begin{matrix} 8 & ^\circ & ' & '' \end{matrix}$
Mean motion in 40000 days	$= 2 - 21 - 18 - 32$
do 1000 days	$= 5 - 14 - 1 - 57$
do 100 days	$= 1 - 22 - 24 - 12$
do 1 day	$= 0 - 0 - 31 - 26$
do 82153 of a day	$= 0 - 0 - 25 - 48$
Position at epoch	$= 7 - 1 - 48 - 54$
Mean Mars at birth moment	$= 5 - 0 - 30 - 49$

# TABLE OF MEAN MOTION OF MARS

Periodic time 686 980 days

Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds
1	0	0	31	26	300	5	7	12	35	50000	9	11	33	10
2	0	1	2	53	400	6	29	36	47	60000	4	1	57	48
3	0	1	34	19	500	8	22	0	59	70000	10	22	17	26
4	0	2	5	16	600	10	14	25	10	80000	5	12	37	4
5	0	2	37	13	700	0	6	49	22	90000	0	2	56	42
6	0	3	8	39	800	1	29	13	34	100000	6	23	16	20
7	0	3	40	6	900	3	21	37	45	200000	1	16	32	40
8	0	4	11	32	1000	5	14	1	57	300000	8	9	49	0
9	0	4	42	59	2000	10	28	3	55	400000	3	3	5	20
10	0	5	14	25	3000	4	12	5	53	500000	9	26	21	40
20	0	10	28	50	4000	9	26	7	51	600000	4	19	37	59
30	0	15	43	16	5000	3	10	9	49	700000	11	12	54	19
40	0	20	57	41	6000	8	24	11	46	800000	6	6	10	39
50	0	26	12	6	7000	2	8	13	44	900000	0	29	26	59
60	1	1	26	31	8000	7	22	15	42	1000000	7	22	43	19
70	1	6	40	56	9000	1	6	17	39					
80	1	11	55	22	10000	6	20	19	38					
90	1	17	9	47	20000	1	10	39	16					
100	1	22	24	12	30000	8	0	58	54					
200	3	14	48	23	40000	2	21	18	32					

Now take the चरप्राणकलांतरदेशांतर बाहुलं 28'—7" (धने) on page 72 Multiply this by the mean daily velocity of Mars viz 31'—26" and divide by 21600 We get 0'—2" (धने). This being + (धने), adding it to the mean Mars already found out viz 5°—0'—30'—51", we get 5°—0'—30'—53". This is mean Mars which has to be used for further calculations

## POSITION OF APSE.

$$\begin{array}{rcl}
 \text{At epoch} & & \begin{array}{c} s \quad \circ \quad ' \quad '' \\ 4-11-18--0 \end{array} \\
 \text{Motion in 112 54 years @ 16 86 per annum is} & & 0--0-31-37 \\
 \text{Adding position of apse at birth} & = & 4-11-49-37 \\
 \text{Mean anomaly} & = & \begin{array}{l} \left\{ \begin{array}{l} 4^s-11'-49''-37'' \text{ minus} \\ 5^s--0'-30'-53'' \end{array} \right. \\ \hline 11^s-11'-18''-44'' = 341^s-15'-44''
 \end{array}
 \end{array}$$

Equation of centre, as defined by us

$$= \frac{e \sin nt}{2} (4 - 5e \cos nt) + \frac{e^3}{12} (13 \sin 3nt - 3 \sin nt)$$

where  $e = 0.9331$

This reduces to, in seconds of arc,

$$9623 (4.05237 - .46655 \cos nt) \sin nt - 728 \sin^3 nt.$$

$$\begin{aligned} \sin 341^\circ - 18' - 44'' &= \sin (360^\circ - 18^\circ - 41' - 16'') \\ &= -\sin 18^\circ - 41' - 16'' \\ &= -.3204017 \end{aligned}$$

$$\begin{aligned} \cos 341^\circ - 18' - 44'' &= \cos (360^\circ - 18^\circ - 41' - 16'') \\ &= \cos 18^\circ - 41' - 16'' \\ &= .9472709 \end{aligned}$$

\* substituting, we get,

$$\begin{aligned} 9623 (4.05237 - .46655 \times .9472709) \times -.3204017 - 728 \times (-.3204017)^3 \\ = -11168'' + 24'' = -11144'' = -(8^\circ - 5' - 44'') \end{aligned}$$

Applying this equation of centre to the mean longitude of Mars we get

$$\begin{array}{rcl} \text{True longitude of Mars} & = & \begin{array}{r} 5^\circ - 0' - 3'' - 53'' \\ \quad \quad \quad 3^\circ - 5' - 44'' \\ \hline 4^\circ - 27' - 25'' - 9'' \end{array} \text{ minus} \end{array}$$

## TO FIND THE RADIUS VECTOR.

$r = \frac{a(1-e^2)}{1-e \cos \theta}$ , where  $a$  is the semi major axis of the ellipse and  $\theta$  is the true anomaly

$$\begin{aligned} \text{Here } a &= 1.5237 \text{ and } \theta = \begin{array}{r} 4^\circ - 11' - 49'' - 37'' \\ \quad \quad \quad 4^\circ - 27' - 25'' - 9'' \\ \hline 11^\circ - 14' - 24'' - 28'' \\ = 344^\circ - 24' - 28'' \end{array} \end{aligned}$$

$$\therefore r = \frac{1.5237 (1+e) (1-e)}{1-e \cos \theta} \text{ and } e \text{ for Mars is } .09331$$

$$\therefore r = \frac{1.5237 \times 1.09331 \times .90669}{1 - .09331 \cos 344^\circ - 24' - 28''}$$

$$\begin{aligned} &= \frac{1.51044}{1 - .09331 \times .196811} = \frac{1.51044}{1 - .08987} = \frac{1.51044}{.91013} \\ &= \underline{1.6596} \end{aligned}$$

The numerator 1.51044 is always same and it is enough if the denominator also is found out and substituted in the formula.

Both the equation of centre and the radius vector can be obtained from the mean anomaly directly, by a reference to the accompanying table. If the mean anomaly were to exceed  $180^\circ$ , then the defect from  $360^\circ$  will be the required argument for purposes of referring to the table. But the equation of centre in the latter case will be negative. The radius vector is always considered positive.

## TABLE OF EQUATION OF CENTRE, RADIUS VECTOR AND HELIOCENTRIC VELOCITY OF MARS.

Arg — Mean anomaly of Mars

Deg	Equation of Centre o ' "			Radius Vector		Hel. Velocity ' "		Deg	Equation of Centre o ' "			Radius Vector		Hel. Velocity ' "	
0	0	0	0	1.6657	26	9		26	4	15	23	1.6536	26	34	
1	0	10	5	1.6657	26	9		27	4	24	41	1.6527	26	36	
2	0	20	9	1.6656	26	10		28	4	33	54	1.6517	26	38	
3	0	30	13	1.6656	26	10		29	4	43	4	1.6507	26	40	
4	0	40	17	1.6654	26	10		30	4	52	10	1.6497	26	42	
5	0	50	20	1.6653	26	11		31	5	1	18	1.6486	26	44	
6	1	0	23	1.6651	26	11		32	5	10	11	1.6475	26	46	
7	1	10	25	1.6649	26	12		33	5	19	2	1.6464	26	48	
8	1	20	25	1.6646	26	12		34	5	27	54	1.6452	26	50	
9	1	30	24	1.6643	26	13		35	5	36	35	1.6440	26	53	
10	1	40	22	1.6640	26	13		36	5	45	19	1.6427	26	56	
11	1	50	20	1.6636	26	14		37	5	53	54	1.6414	26	58	
12	2	0	17	1.6632	26	14		38	6	2	25	1.6401	27	1	
13	2	10	13	1.6627	26	16		39	6	10	50	1.6388	27	4	
14	2	20	6	1.6622	26	17		40	6	19	10	1.6374	27	6	
15	2	29	56	1.6617	26	18		41	6	27	25	1.6360	27	9	
16	2	39	43	1.6611	26	19		42	6	35	35	1.6346	27	12	
17	2	49	28	1.6605	26	21		43	6	43	39	1.6331	27	15	
18	2	59	10	1.6599	26	22		44	6	51	40	1.6316	27	18	
19	3	8	51	1.6592	26	23		45	6	59	30	1.6301	27	21	
20	3	18	30	1.6585	26	25		46	7	7	17	1.6286	27	25	
21	3	28	51	1.6578	26	26		47	7	14	57	1.6270	27	28	
22	3	37	38	1.6570	26	27		48	7	22	32	1.6254	27	32	
23	3	47	12	1.6562	26	29		49	7	30	0	1.6238	27	35	
24	3	56	37	1.6554	26	30		50	7	37	22	1.6221	27	38	
25	4	6	2	1.6545	26	32		51	7	44	38	1.6204	27	42	

52	7	51	45	1 6187	27	45	96	10	44	42	1 5227	31	24
53	7	58	48	1 6170	27	49	97	10	44	46	1 5200	31	30
54	8	5	13	1 6152	27	53	98	10	44	89	1 5173	31	36
55	8	12	31	1 6134	27	57	99	10	44	18	1 5146	31	43
56	8	19	10	1 6116	28	1	100	10	43	45	1 5121	31	49
57	8	25	46	1 6097	28	5	101	10	43	0	1 5096	31	56
58	8	32	9	1 6078	28	8	102	10	42	6	1 5071	32	3
59	8	38	34	1 6059	28	12	103	10	40	56	1 5046	32	10
60	8	44	33	1 6040	28	16	104	10	39	34	1 5021	32	17
61	8	50	41	1 6020	28	20	105	10	38	1	1 4997	32	24
62	8	56	34	1 6000	28	24	106	10	36	16	1 4972	32	30
63	9	2	23	1 5980	28	28	107	10	34	16	1 4947	32	36
64	9	8	2	1 5960	28	33	108	10	32	7	1 4923	32	42
65	9	13	31	1 5940	28	37	109	10	29	42	1 4893	32	48
66	9	18	54	1 5919	28	42	110	10	27	7	1 4873	32	54
67	9	24	6	1 5893	28	47	111	10	24	19	1 4849	33	1
68	9	29	12	1 5877	28	51	112	10	21	18	1 4824	33	7
69	9	34	8	1 5856	28	56	113	10	18	3	1 4800	33	14
70	9	38	55	1 5834	29	1	114	10	14	38	1 4776	33	21
71	9	43	31	1 5812	29	6	115	10	10	59	1 4752	33	28
72	9	48	2	1 5790	29	11	116	10	7	8	1 4728	33	35
73	9	52	19	1 5768	29	16	117	10	3	4	1 4704	33	42
74	9	56	31	1 5746	29	20	118	9	58	44	1 4680	33	49
75	10	0	31	1 5723	29	25	119	9	54	17	1 4656	33	56
76	10	4	22	1 5701	29	30	120	9	49	58	1 4633	34	4
77	10	8	4	1 5678	29	35	121	9	44	49	1 4609	34	11
78	10	11	36	1 5655	29	40	122	9	39	34	1 4586	34	19
79	10	14	54	1 5632	29	45	123	9	34	14	1 4563	34	26
80	10	18	5	1 5609	29	50	124	9	28	41	1 4540	34	32
81	10	21	8	1 5585	29	56	125	9	22	57	1 4518	34	37
82	10	23	59	1 5562	30	1	126	9	17	2	1 4496	34	42
83	10	26	37	1 5533	30	6	127	9	10	53	1 4475	34	48
84	10	29	7	1 5514	30	12	128	9	4	32	1 4452	34	54
85	10	31	27	1 5490	30	18	129	8	57	59	1 4431	34	59
86	10	33	36	1 5466	30	23	130	8	51	13	1 4409	35	5
87	10	35	32	1 5442	30	29	131	8	44	16	1 4383	35	11
88	10	37	19	1 5418	30	35	132	8	37	7	1 4367	35	17
89	10	38	56	1 5394	30	41	133	8	29	47	1 4346	35	23
90	10	40	18	1 5369	30	47	134	8	22	12	1 4326	35	29
91	10	41	33	1 5345	30	53	135	8	14	30	1 4306	35	35
92	10	42	33	1 5320	31	0	136	8	6	36	1 4286	35	41
93	10	43	22	1 5295	31	6	137	7	58	29	1 4266	35	47
94	10	44	1	1 5272	31	12	138	7	50	10	1 4247	35	53
95	10	44	28	1 5249	31	18	139	7	41	42	1 4223	35	59



140	7	38	8	1 4210	36	3	161	3	55	4	1.3910	37	38
141	7	24	11	1 4192	36	9	162	3	43	16	1 3900	37	41
142	7	15	11	1 4174	36	14	163	3	31	22	1 3891	37	43
143	7	5	59	1 4156	36	20	164	3	19	24	1 3883	37	45
144	6	56	38	1 4139	36	26	165	3	7	16	1 3875	37	46
145	6	47	7	1 4122	36	31	166	2	55	12	1 3866	37	48
146	6	37	26	1 4106	36	36	167	2	43	4	1 3861	37	50
147	6	27	35	1 4090	36	42	168	2	30	43	1 3854	37	52
148	6	17	33	1 4074	36	47	169	2	18	27	1 3848	37	54
149	6	7	26	1 4059	36	52	170	2	6	4	1 3843	37	55
150	5	57	8	1 4044	36	56	171	1	53	33	1 3838	37	57
151	5	46	40	1 4030	37	0	172	1	41	7	1 3834	37	59
152	5	36	5	1 4016	37	4	173	1	28	34	1 3830	38	1
153	5	25	21	1 4002	37	8	174	1	15	58	1 3826	38	3
154	5	14	29	1 3989	37	12	175	1	3	21	1 3823	38	3
155	5	3	29	1 3977	37	16	176	0	50	43	1 3821	38	4
156	4	52	20	1 3965	37	19	177	0	38	3	1 3819	38	4
157	4	41	7	1 3953	37	23	178	0	25	24	1 3818	38	5
158	4	29	47	1 3941	37	27	179	0	12	42	1 3817	38	6
159	4	18	20	1 3930	37	30	180	0	0	0	1 3816	38	6
160	4	6	48	1 3920	37	34							

If mean anomaly is more than  $180^\circ$ , then its defect from  $360^\circ$  will be the argument for referring to the tables but the equation of centre is negative

In the present instance mean anomaly is  $11^\circ-11'-18''-44''$  or  $341^\circ-18'-44''$ .

The argument to refer the tables is  $18^\circ-41'-16''$ .

For  $18^\circ$ , equation is  $2^\circ-59'-10''$  and for  $19^\circ$  equation is  $3^\circ-8'-51''$

$\therefore$  for  $18^\circ-41'-16''$ , it is  $3^\circ-5'-49''$  As this equation of centre is negative, subtracting it from the mean Mars  $5^\circ-0'-30'-53''$  we get  $4^\circ-27'-25'-4''$  Radius vector is 16596 Heliocentric Velocity is  $26'-23''$  as read from the tables

## REDUCTION.

Position of Mars' node at epoch =  $0^\circ-26'-54'-48''$

Annual motion =  $-22''.74$

• Motion in 11254 years =  $-22''.74 \times 11254 = -42'-39''$

$\therefore$  Position of Node at birth =  $0^\circ-26'-12'-9''$

$$\therefore \text{Nodal distance of planet (PN)} = \begin{cases} 4^s-27^m-25^s-9'' \text{ minus} \\ 0^s-26^m-12^s-9'' \\ \hline = 4^s-1^m-13^s-0'' \\ = 121^m-13^s-0'' \end{cases}$$

Now  $\tan NM = \cos i \tan PN$

$$\begin{aligned} L \tan NM &= L \cos i + L \tan PN - 10 \\ &= 9.9997736 + 10.2175136 - 10 \\ &= 10.2172872 \end{aligned}$$

$$\therefore NM = 180^m \text{ minus } 58^m-46^s-12''$$

$$\left. \begin{aligned} \odot N &= 4^s-1^m-13^s-48'' \\ &= 0^s-26^m-12^s-9'' \end{aligned} \right\} \therefore \odot M = 4^s-27^m-25^s-57''$$

This is the True H L referred to the ecliptic. This correction due to the inclination of the orbit to the ecliptic is very small. It can be found with the help of the reduction table of Moon with the Nodal distance as the argument, the reduction thus got is multiplied by a multiplier 13, which will be the reduction for Mars.

In this instance reduction from Moon's table for a Nodal distance  $121^m-13^s-0''$  is (+)  $6^s-9''$ . Applying the multiplier 13 we get reduction for Mars as 48 (+). This is added, to the H L of Mars. It will be  $(4^s-27^m-25^s-9'') + 48'' = 4^s-27^m-25^s-57''$

## HELIOCENTRIC LATITUDE OF PLANET (Arc PM)

That portion of the vertical passing thro' the pole of the orbit of the planet and the planet intercepted by the orbit and the ecliptic, it is called the Latitude. It is N or S according as the Nodal distance is > or <  $180^m$

$$\begin{aligned} \sin PM &= \cos (90^m-i) \cos (90^m-PN) \\ &= \sin i \sin PN \end{aligned}$$

$$\begin{aligned} L \sin PM &= L \sin i + L \sin PN - 10 \\ &= 8.5089736 + 9.9320266 - 10 \\ &= 8.4410022 \therefore PM = 1^m-35^s'N \end{aligned}$$

We have now found out all the various coordinates which would be observed if one were stationed at the centre of the sun. We will be presently transferring them to the centre of the earth.

In figure of page 126 we have

SP = 1.6596	}	SM = SP cos PSM
$\angle PSM = 1^m-35^s'$		= 1.6596 cos $1^m-35^s'$
$\angle \odot SM = 147^m-25^s-57''$		= 1.6596 $\times$ .9962
SE = 1.01646		= 1.6531
$\angle TSE = \text{sun True Longitude} + 180^m$		= $2^m-29^s-21^s-22''$ plus
		$6^m-0^s-0^s-0''$

$$= 269^{\circ}-21'-22''$$

$$\therefore \widehat{MSE} = \begin{cases} 147^{\circ}-25'-57'' \text{ minus} \\ 269^{\circ}-21'-22'' \\ \hline 238^{\circ}-4'-35'' \end{cases}$$

$$\begin{aligned} \text{Now } EM^2 &= SM^2 + SE^2 - 2SM \cdot SE \cos \widehat{MSE} \\ &= (1.653)^2 + (1.016)^2 - 2 \times 1.653 \times 1.016 \cos 238^{\circ}-4'-35'' \\ &= 2.7326 + 1.032 - 3.359 \times \cos 58^{\circ}-4'-35'' \\ &= 3.7646 + 3.359 \times .5286 \\ &= 3.7646 + 1.7755 = 5.5401 \end{aligned}$$

$$\therefore EM = \sqrt{5.5401} = 2.3537$$

$$\text{We have } \frac{EM}{\sin \angle ESM} = \frac{SE}{\sin \angle SME}$$

$$\begin{aligned} \therefore \sin \angle SME &= \frac{SE}{EM} \sin \angle ESM \\ &= \frac{1.01646}{2.3537} \times \sin 238^{\circ}-4'-35'' \\ &= \frac{1.01646}{2.3537} \times \sin 58^{\circ}-4'-35'' \\ &= \frac{1.01646}{2.3537} \times .8487538 \\ &= .3664372 \end{aligned}$$

$$\therefore \angle SME = -(20^{\circ}-29'-46'')$$

Geocentric Longitude

$$\begin{aligned} (\angle^{\circ}EM) &= \angle^{\circ}TSM + \angle SME \\ &= (147^{\circ} \ 25' - 57'') + (-(21^{\circ}-29'-46'')) \\ &= 125^{\circ}-56'-11'' = \underline{4^{\circ}-5^{\circ}-56'-11''} \end{aligned}$$

TO FIND THE GEOCENTRIC LATITUDE.

$$PM = SP \sin \angle PSM = EM \tan \angle PEM$$

$$\therefore \tan \angle PEM = \frac{SP}{EM} \sin \angle PSM = \frac{1.653}{2.354} \times .027631$$

(for  $\angle PSM = \text{Heliocentric latitude} \approx 1^{\circ}35' \text{ N}$ )

$$\therefore \tan \angle PEM = .019445 \therefore PEM = \underline{1^{\circ}-7' \text{ N}}$$

TO FIND THE GEOCENTRIC VELOCITY

$$\sin \angle SME = \frac{SE}{EM} \sin \angle ESM$$

$$\therefore \text{Rate of change of } \angle \text{SME} = \frac{\text{SE} \cos \angle \text{ESM}}{\text{EM} \cos \angle \text{SME}} \times \text{rate of change of } \angle \text{ESM}$$

$$= \frac{1\ 01646}{2\ 354} \times \frac{-5286}{+9307} \times \left\{ (26' - 23'') - (57' - 12'') \right\}$$

$$= 7' - 48''$$

$\therefore$  Rate of change of  $\angle \text{TSM} + \text{Rate of change of } \angle \text{SME}$   
 $= \text{Rate of change of } \angle \text{TEM}$

(i)  $e) (26' - 23'') + (7' - 48'') = \text{velocity of } \angle \text{TEM}$

(i)  $e) 34' - 11'' = \text{geocentric velocity of the planet}$

Hence the planet is not retrograde

S

[The rate of change of  $\angle \text{ELM}$  is got thus  $\angle \text{ESM} = \angle \text{TSM} - \angle \text{TSE}$

$\therefore$  Rate of change of  $\angle \text{ESM} = \text{rate of change of } \angle \text{TSM} - \text{that of } \angle \text{TSE}$   
 $= \text{Heliocentric velocity of planet} - \text{Heliocentric velocity of Earth}]$

## Chapter XIV.

### MERCURY.

#### ELEMENTS

1	Mean longitude at epoch	$2^{\circ} - 29^{\circ} - 19' - 17''$
2	Mean Hel longitude of Apse	$7^{\circ} - 23^{\circ} - 17' - 49''$
3	Mean Hel longitude of Node	$0^{\circ} - 24^{\circ} - 59' - 38''$
4	Length of semi major axis	3871
5	Excentricity of orbit	20561
6	Inclination of orbit to ecliptic	$7^{\circ} - 0'$
7.	Periodic time	87 969 days
8	Annual motion of Apse $6^{\circ} 14 (+)$	
9	Annual motion of Nodes $6^{\circ} 52 (-)$	
No of days elapsed from epoch to the moment of Birth = 41101 82153 days		
Periodic time = 87 969 days		

$$\therefore \text{No of revolutions} = \frac{41101\ 82153}{87\ 969}$$

$$= 467\ 23075$$

Converting 2375 of a revolution to signs etc, we get  $2^s-23^m-4^s-12^s$ , to which if epoch mean position viz,  $2^s-29^m-19^s-17^s$  be added, we get mean longitude of Mercury as  $5-22^m-23^s-29^s$ .

## HINDU METHOD.

Divide the number of days by 88 The quotient will be the number of revolutions With the balance get signs etc (A) Again divide the number of days by 708, the quotient will be degrees and the balance if any reduced to minutes and seconds (B).

The sum of the two (A+B) will be the mean motion of Mercury for the number of days taken. Empirical correction is  $1751'-35''$  for 1000 000 days additive.

$$\begin{array}{rcl}
 \text{(A)} & \frac{4110182153}{88} = 167 \frac{582153}{58} & = 0-23-18-55 \\
 \text{(B)} & \frac{4110182153}{708} = 58 \frac{3782153}{708} & = 0-58-3-12 \\
 & \text{Adding up, we get} & 2-21-52-7 \\
 & \text{Empirical correction} & = \frac{1751'-35'' \times 41102}{1000000} \\
 & & = 1^s-12'-0'' \\
 & \text{Corrected mean motion} & = 2-23-4-7 \\
 & \text{Epoch mean position} & = 2-29-19-17 \\
 & \therefore \text{Mean longitude of Mercury at birth} & = 5-22-23-24
 \end{array}$$

Now this is found to tally very much with the result of the long division. Then take the net correction to be applied due to the place of birth etc चरप्राणभ्रमंतरदेशांतरबाहुफलं  $28'-7(+)$  Multiply by Mercury's mean daily velocity  $245'-32$  and divide by 216000 The result will be the correction to be applied It is  $\frac{245'-32'' \times 28'-7''}{216000} = 19''(+)$

Applying this to the mean longitude already arrived at viz  $5^s-22^m-23^s-29^s$  we get  $5^s-22^m-23^s-48^s$  If applied to the Hindu result will be  $5^s-22^m-23^s-13^s$  We shall however adhere to the result of the long division method

# TABLE OF MEAN MOTION OF MERCURY

Periodic time = 87 969 days

Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds
1	0	4	5	32	300	4	27	42	19	50000	4	17	32	9
2	0	8	11	5	400	6	16	56	25	60000	0	21	2	34
3	0	12	16	37	500	8	6	10	32	70000	8	24	33	0
4	0	16	22	10	600	9	25	24	38	80000	4	28	3	26
5	0	20	27	42	700	11	14	38	44	90000	1	1	38	52
6	0	24	33	14	800	1	3	52	50	100000	9	5	4	17
7	0	28	38	47	900	2	23	6	57	200000	6	10	8	34
8	1	2	44	19	1000	4	12	21	3	300000	3	15	12	50
9	1	6	49	52	2000	8	24	42	5	400000	0	20	17	7
10	1	10	55	24	3000	1	7	3	8	500000	9	25	21	24
20	2	21	50	48	4000	5	19	24	11	600000	7	0	25	41
30	4	2	46	12	5000	10	1	45	14	700000	4	5	29	57
40	5	13	41	36	6000	2	14	6	16	800000	1	10	34	14
50	6	24	37	1	7000	6	26	27	19	900000	10	15	38	31
60	8	5	32	25	8000	11	8	48	22	1000000	7	20	42	48
70	9	16	27	49	9000	3	21	9	24					
80	10	27	23	13	10000	8	3	30	26					
90	0	8	18	37	20000	4	7	4	52					
100	1	19	14	6	30000	0	10	31	17					
200	3	8	28	13	40000	8	14	1	43					

## IN THE EXAMPLE

40000 days	8-14-1-43
1000 days	4-12-21-3
100 days	1-19-11-6
1 days	0-1-5-32
82153 day	0-3-21-43
	2-23-4-7
Epoch mean position	2-29-19-17
Corrections due to observer	19 †
∴ Mean longitude of Mercury	5-22-23-43

32	10	15	43	4582	173	16	76	20	57	3	4204	205	21
33	10	33	32	4577	173	45	77	21	7	16	4192	206	26
34	10	51	16	4572	174	16	78	21	17	14	4180	207	31
35	11	8	52	4566	174	48	79	21	26	57	4168	208	40
36	11	26	22	4560	175	20	80	21	36	22	4156	209	50
37	11	43	44	4554	175	53	81	21	45	33	4144	211	0
38	12	0	56	4548	176	26	82	21	54	27	4132	212	14
39	12	18	2	4541	177	0	83	22	3	2	4120	213	29
40	12	35	1	4534	177	34	84	22	11	21	4107	214	44
41	12	51	49	4527	178	8	85	22	19	22	4095	216	3
42	13	8	29	4521	178	43	86	22	27	4	4082	217	22
43	13	25	0	4514	179	19	87	22	34	28	4069	218	42
44	13	41	27	4507	179	55	88	22	41	32	4056	220	7
45	13	57	42	4500	180	32	89	22	48	17	4043	221	32
46	14	13	50	4493	181	10	90	22	54	42	4030	222	58
47	14	29	47	4485	181	48	91	23	0	46	4017	224	28
48	14	45	37	4477	182	26	92	23	6	29	4004	225	58
49	15	1	16	4469	183	5	93	23	11	51	3991	227	29
50	15	16	47	4461	183	45	94	23	16	50	3978	229	5
51	15	32	8	4453	184	24	95	23	21	28	3964	230	42
52	15	47	20	4445	185	4	96	23	25	43	3950	232	19
53	16	2	22	4437	185	44	97	23	29	35	3937	234	2
54	16	17	13	4428	186	25	98	23	32	2	3923	235	45
55	16	31	55	4419	187	8	99	23	36	5	3909	237	27
56	16	46	28	4410	187	52	100	23	38	44	3895	239	15
57	17	0	50	4401	188	36	101	23	40	57	3882	241	4
58	17	15	3	4392	189	21	102	23	44	44	3868	242	53
59	17	29	1	4383	190	6	103	23	44	5	3854	244	48
60	17	42	53	4373	190	52	104	23	46	1	3840	246	42
61	17	56	31	4364	191	39	105	23	47	28	3826	248	37
62	18	10	1	4354	192	27	106	23	45	26	3812	250	37
63	18	23	17	4344	193	15	107	23	44	58	3798	252	37
64	18	36	22	4334	194	5	108	23	44	1	3784	254	37
65	18	49	16	4324	194	55	109	23	42	34	3769	256	42
66	19	1	58	4314	195	45	110	23	40	37	3755	258	47
67	19	14	26	4304	196	39	111	23	38	11	3741	260	51
68	19	26	44	4293	197	33	112	23	35	15	3727	263	2
69	19	38	49	4282	198	26	113	23	31	47	3713	265	12
70	19	50	40	4271	199	23	114	23	27	50	3699	267	22
71	20	2	19	4260	200	20	115	23	23	20	3684	269	36
72	20	13	44	4249	201	16	116	23	18	17	3670	271	50
73	20	24	54	4238	202	16	117	23	12	44	3656	274	3
74	20	35	52	4227	203	16	118	23	6	37	3642	276	22
75	20	46	34	4215	204	17	119	22	9	57	3628	278	41

120	22 52 45	.8614	281 0	151 14 24 54	.8226	851 9
121	22 44 59	.8600	289 21	152 13 59 49	.8216	853 2
122	22 36 35	.8586	285 41	153 13 34 14	.8207	854 54
123	22 27 41	.8572	288 1	154 13 8 13	.8198	856 33
124	22 18 13	.8558	290 24	155 12 41 44	.8189	857 12
125	22 8 6	.8544	292 47	156 12 14 52	.8181	859 51
126	21 57 31	.8530	295 10	157 11 47 31	.8173	861 21
127	21 46 17	.8516	297 34	158 11 19 47	.8165	862 51
128	21 34 29	.8502	299 57	159 10 51 42	.8157	864 21
129	21 22 6	.8489	302 20	160 10 23 13	.8150	865 40
130	21 9 8	.8475	304 45	161 9 54 22	.8143	866 59
131	20 55 34	.8462	307 10	162 9 25 10	.8136	868 18
132	20 41 26	.8449	309 34	163 8 55 38	.8130	869 22
133	20 26 43	.8436	312 36	164 8 25 47	.8124	870 27
134	20 11 22	.8423	315 38	165 7 55 40	.8118	871 32
135	19 55 30	.8410	318 39	166 7 25 14	.8112	872 27
136	19 38 59	.8397	320 19	167 6 54 31	.8107	873 21
137	19 21 52	.8384	321 59	168 6 23 37	.8102	874 15
138	19 4 19	.8372	323 39	169 5 52 34	.8098	874 55
139	18 46 7	.8359	325 55	170 5 21 7	.8094	875 35
140	18 27 21	.8347	328 11	171 4 49 33	.8091	876 16
141	18 7 59	.8335	330 27	172 4 17 50	.8088	876 42
142	17 48 5	.8323	332 37	173 3 45 57	.8085	877 8
143	17 27 39	.8312	334 47	174 3 13 53	.8082	877 33
144	17 6 40	.8301	336 57	175 2 41 47	.8080	877 41
145	16 45 6	.8289	339 5	176 2 9 32	.8078	877 48
146	16 23 1	.8278	341 13	177 1 37 14	.8077	877 55
147	16 0 23	.8267	343 20	178 1 4 51	.8076	877 57
148	15 37 16	.8256	345 19	179 0 32 26	.8075	877 59
149	15 13 40	.8246	347 18	180 0 0 0	.8075	878 2
150	14 49 31	.8236	349 16			

In the example taken mean anomaly is  $61^{\circ}-5'-32'$  Entering the table with this argument, we have for  $61^{\circ}$  equation of centre as  $17^{\circ}-56'-31''$  and increase for  $1^{\circ}$  as  $13'-30''$

$$\therefore \text{Equation for } 61^{\circ}-5'-32'' = 17^{\circ}-57'-46''$$

This is positive as the mean anomaly is within  $180^{\circ}$ . Adding this to the mean longitude, we have  $5^{\circ}-22^{\circ}-23'-48''$  plus  $0^{\circ}-17^{\circ}-57'-46'' = 6^{\circ}-10^{\circ}-21'-34''$ ; so also, we get radius vector as .4363 and velocity as  $191'-45''$ . This velocity is subject to the following correction when less than the mean:—Find the difference between the velocity got and the mean velocity  $245'-32''$ . Multiply it by  $\frac{7}{25}$ . If the previously got velocity is less than the mean velocity, subtract the rectified velocity from the mean velocity



The result will be the correct velocity. In the example, difference between the velocity got and the mean velocity is 53'—47". Applying the multiplier we get

$$\frac{27}{100} \times 53.8 = 58.1 \text{ or } 58'—6''$$

subtracting it from the mean velocity we get 187'—26'' as the velocity. There is a slight difference between this and that obtained trigonometrically, and it is due to the high value of the mean velocity. But this will not create any appreciable difference in the ultimate geocentric velocity.

## REDUCTION.

Position of Node at epoch	0°—24'—53'—38"
Annual motion of Node	= —6" 82
∴ Motion in 112 54 years	= —6" 82 × 112 54
	= —12'—48"
∴ Position of Node at birth	= 0°—24'—40'—50"
∴ Distance of planet in its own orbit from the Node is	6°—10'—21'—37" minus
	0°—24'—40'—50"
	5°—15'—40'—47"
NP	= 5°—15'—40'—47"

$$\begin{aligned} \therefore \tan NM &= \cos i \tan NP \\ &= \cos 7^\circ \tan 165^\circ—40'—47'' \end{aligned}$$

For  $i$  in the case of orbit of Mercury is 7

$$\begin{aligned} \therefore \tan NM &= -\cos 7^\circ \tan 14^\circ—19'—13'' \\ \text{from which } NM &= 180^\circ—0'—0'' \text{ minus} \\ &\quad \underline{14^\circ—13'—4''} \\ &\quad \underline{\hspace{1.5cm}} \\ &\quad 165^\circ—46'—56'' \end{aligned}$$

∴ Heliocentric longitude corrected for reduction is 6°—10'—27'—46", for  $\Upsilon M = \Upsilon N + NM$  from the figure of page 128

The reduction can also be found out with the help of the Moon's reduction table and applying the multiplier 1851, which will give the reduction to be made for Mercury

## HELIOCENTRIC LATITUDE.

$$\begin{aligned} \sin PM &= \sin i \sin NP \\ &= \sin 7^\circ \sin 165^\circ—40'—47'' \\ \therefore L \sin PM &= L \sin 7^\circ + L \sin 165^\circ—40'—47'' = 10 \end{aligned}$$

$$\begin{array}{r}
= 9\ 0858945+ \\
9\ 3933885- \\
10 \\
\hline
= 8\ 4792830 \\
\hline
\end{array}$$

$\therefore PM = 1^\circ - 43' - 40''$  N as Nodal distance is within  $180^\circ$

$$\begin{aligned}
\therefore SM &= SP \cos \hat{PSM} = 4361 \cos 1^\circ - 43' - 40'' \\
&= 4361 \times .9995454 = 4359
\end{aligned}$$

$SE = 1\ 01646$  (see under Earth's radius vector)

$$\begin{aligned}
\hat{MSE} &= \hat{TSM} - \hat{TSE} + 180^\circ \\
&= (190^\circ - 27' - 46'') - (89^\circ - 12' - 2'') + 180^\circ \\
&= 281^\circ - 6' - 24''
\end{aligned}$$

$$\begin{aligned}
ME^2 &= MS^2 + SE^2 - 2SE \cdot SM \cos 281^\circ - 6' - 24'' \\
&= (.4359)^2 + (1\ 01646)^2 - .8718 \times 1\ 01646 \times \cos (281^\circ - 6' - 24'') \\
&= .19001 + 1\ 032 - .17066, \text{ [for } \cos 281^\circ - 6' = \cos 78^\circ - 54' \\
&\qquad\qquad\qquad = \sin 10^\circ - 6' = .1996] \\
&= 1.05145
\end{aligned}$$

$$\therefore ME = \sqrt{1.05145} = 1.0251$$

$$\begin{aligned}
\text{We have } \sin \hat{SME} &= \frac{SE}{ME} \sin \hat{MSE} \\
&= \frac{1\ 01646}{1.0251} \sin 281^\circ - 6' - 24'' \\
&= \frac{1\ 01646}{1.0251} \times -.9812708 = -.973 \\
&= \sin (-76^\circ - 39' - 20'') \therefore \hat{SME} = -(76^\circ - 39' - 20'')
\end{aligned}$$

$$\begin{aligned}
\therefore \hat{EM} &= \hat{TSM} + \hat{SME} \\
&= (6^\circ - 10' - 27' - 46'') + (-76^\circ - 39' - 20'') \\
&= 6^\circ - 10' - 27' - 46'' \text{ minus} \\
&\quad 2^\circ - 16' - 39' - 20'' \\
&\hline
&\quad 3^\circ - 23' - 47' - 26'' \\
&\hline
\end{aligned}$$

This is the true geocentric longitude of Mercury

The principle in taking the correct value of  $\hat{SME}$  has been observed here For,  $\angle MSE = 281^\circ - 6' - 24''$  This being greater than  $180^\circ$ , subtract from  $360^\circ$  We get  $78^\circ - 53' - 36''$   $\angle SME$  has been found out to be  $76^\circ - 39' - 20''$ . Then  $\angle MES$  will be  $180^\circ - (76^\circ - 39' - 20'' + 78^\circ - 53' - 36'')$  which is equal to  $24^\circ - 27' - 4''$  We require that  $\angle SME$  should be greater than  $\angle MES$  in magnitude which is found to be true Hence the value

of  $\angle SME$  taken by us is correct. If we had not had this condition satisfied we should have taken the supplement of the  $\angle SME$  arrived at

## GEOCENTRIC LATITUDE.

$$\tan \angle PSM (\text{Hel lat}) = \frac{PM}{SM} \quad (1) \quad PM = SM \tan \hat{PSM}$$

$$\text{but } \frac{PM}{EM} = \tan \text{Geo latitude}$$

$$\therefore \tan (\text{Geo latitude}) = \frac{SM}{EM} \tan \hat{PSM}$$

$$= \frac{.4359}{1.0251} \times \tan 1^\circ 43' 40'' = \frac{.4359}{1.0251} \times .0391646$$

$$= .0128268 \quad \therefore \text{Geo latitude} = 0^\circ 44' 5''$$

## GEOCENTRIC VELOCITY.

This = Velocity of  $\hat{P}SM$  + Velocity of  $\angle SME$

but rate of change of  $\angle SME$  or velocity of  $\angle SME$

$$= \left[ \frac{SE}{ME} \frac{\cos MSE}{\cos SME} - \frac{SM}{ME^2} \frac{SE}{\cos SME} \sin^2 MSE \right] \times \left( \text{rate of change of } \angle MSE \right)$$

$$= \left[ \frac{1.01646}{1.0251} \times \frac{.1976}{.2306} - \frac{4.59 \times 1.01646}{(1.0251)^3} \times \frac{(9813)^2}{.2306} \right] \times (125' - 40'')$$

$$= (82825 - 17175) \times (125' - 40'')$$

$$= -.8892 \times (125' - 40'') = -(111' - \frac{44}{7}'')$$

$$\therefore \text{Geocentric Velocity} = \text{Velocity of } \angle PSM + \text{Velocity of } \angle SME$$

$$= (182' - 51'') - (111' - \frac{44}{7}'') = (65' - 17'')$$

It should be observed that in the expression written here above for the velocity of  $\angle SME$  only the first term viz  $\frac{SE}{ME} \frac{\cos MSE}{\cos SME}$  was given under Mars as the other term varies numerically between the values 1 to 25 nearly, taking even the most favourable conditions. It is desirable that the elaborate formula only should be used but except for Mars among the major planets the first term is quite sufficient. For Mercury and Venus the entire expression has to be used always. It was therefore why the second term was not spoken of at all under Mars. Even it had been taken the difference in the final geocentric velocity would have been very negligible say 3'. At certain times near the stationery points where the bodies appear to move with zero velocity it would be worth while to use the entire expression.

Considering the labouriousness of the working even the ephemeris give only the daily longitudes of planets and the velocity is got as the

difference of the longitudes on any consecutive days but it will fail to give the exact geocentric velocity at a particular instant unless it be with the above detailed formula

## Chapter XV.

### JUPITER.

#### ELEMENTS

1	Mean longitude of Jupiter at epoch	2—0—50—57
2	Mean Hel longitude of Apse	5—20—4—37
3	Mean Hel longitude of Node	2—17—22—44
4	Length of semi major axis	5 2028
5	Excentricity of orbit	= 04833
6	Inclination of orbit to ecliptic	= 1°—19
7	Annual motion of Apse	= 6 63 (+)
8	Annual motion of Node	= 14" 4 (—)
9	Periodic time	= 4332 585 days
	No of days from epoch	—41101 82153
	Periodic time	= 4332 585
	No of revolutions	= $\frac{41101\ 82153}{4332\ 585}$
		= 9 48667

Converting decimal portion of a revolution to signs etc we get 5<sup>s</sup>—25°—12—4 to which if the epoch mean position viz 2<sup>s</sup>—0°—50—57" be added we get 7<sup>s</sup>—26°—3—1" as the mean longitude of Jupiter

#### HINDU METHOD.

Divide the number of days from epoch by 360 The quotient will be signs With the remainder we get degrees etc Let this be A Again divide the number of days by 67 The quotient will be minutes and the balance if any is converted to seconds Let this be B Then (A—B) will be the mean motion of Jupiter The Empirical correction is 401—33" for 1 000 000 days additive

$$(A) \frac{11101 \ 82153}{160} = 114 \text{ } ^{\circ} 5' 9'' = 6 \text{ } ^{\circ} 5' 9'' \text{ (leaving off number revolutions)}$$

$$(B) \frac{11101 \ 82153}{67} = 613 \frac{30 \ 82153}{67} \text{ minutes of arc}$$

$$= 10^{\circ} 13' 28''$$

$$A-B = 5^{\circ} 21' 55''$$

$$\text{Empirical correction} = \frac{41101 \times 101 \ 55}{1000000}$$

$$= 16 \text{ } ^{\circ} 30' (+)$$

$$\text{Rectified motion} = 5^{\circ} 25' 12''$$

$$\text{Epoch position} = 2^{\circ} 0' 50''$$

$$\text{Correct mean longitude of Jupiter at birth} = 7^{\circ} 26' 3''$$

The चरप्राणदशांतरगह्वर correction is got by multiplying the net result  $28-7 (+)$  by Jupiter's mean velocity  $4-59$  and dividing by 216000. In the present example it is  $\frac{28-7'' \times 4-59}{21600} = 4$  of a second. As this is very negligible the previously arrived at mean longitude is sufficiently accurate.

### EXAMPLE

40000 days	$2^{\circ} 23' 39''$
1000 days	$2^{\circ} 23' 5''$
100 days	$0^{\circ} 8' 18''$
1 days	$0^{\circ} 0' 4''$
82153 day	$0^{\circ} 0' 4''$
Mean motion	$5^{\circ} 25' 12''$
Epoch	$2^{\circ} 0' 50''$
Mean longitude	$7^{\circ} 26' 3''$

# TABLE OF JUPITERS' MEAN MOTION.

Periodic time of Jupiter = 4332 585 days

Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds
1	0	0	4	59	300	0	24	55	39	50000	6	14	33	49
2	0	0	9	58	400	1	3	14	12	60000	10	5	28	35
3	0	0	14	57	500	1	11	32	45	70000	1	26	23	21
4	0	0	19	56	600	1	19	51	17	80000	5	17	18	6
5	0	0	24	55	700	1	28	9	50	90000	9	8	12	52
6	0	0	29	55	800	2	6	28	23	100000	0	29	7	38
7	0	0	34	54	900	2	14	46	56	200000	1	28	15	16
8	0	0	39	53	1000	2	23	5	29	300000	2	27	22	54
9	0	0	44	52	2000	5	16	10	57	400000	3	26	30	32
10	0	0	49	51	3000	8	9	16	26	500000	4	25	38	10
20	0	1	39	42	4000	11	2	21	54	600000	5	24	45	48
30	0	2	29	34	5000	1	25	27	23	700000	6	23	53	26
40	0	3	19	25	6000	4	18	32	52	800000	7	23	1	4
50	0	4	9	16	7000	7	11	38	20	900000	8	22	8	42
60	0	4	59	7	8000	10	4	43	49	1000000	9	21	16	20
70	0	5	48	58	9000	0	27	49	17					
80	0	6	38	50	10000	3	20	54	46					
90	0	7	28	41	20000	7	11	49	32					
100	0	8	18	33	30000	11	2	44	17					
200	0	16	37	6	40000	2	23	39	3					

## CORRECTION TO MEAN JUPITER DUE TO SATURN'S ATTRACTION.

As Jupiter and Saturn are the two biggest planets of the Solar system and as they are also neighbours to each other the natural attraction between each other affects the mean position of either which cannot be overlooked if a correct value of their true longitude were required

Jupiter's correction will be

$$-20.8 \sin t (5s-2j) - 1.3783 \sin (H-h)$$

$$+ 3.405 \sin 2(H-h) + 283 \sin 3(H-h)$$

where  $t$  is the number of years passed after 1558 March and  $(H-h) = 18^\circ 129 \times (t-211.75) - (41^\circ - 11')$  and  $(5s-2j) = 4074926$

In the present instance the year of birth is 1912 July wherefore  $t = 1912 \text{ July} - 1558 \text{ March} = 354.33$

$$\begin{aligned}
& \therefore \text{Jupiters' correction will be} \\
& = 20'8 \times \sin (354.33 \times .4074926) \\
& = 1.3783 \sin \{18^\circ.129 \times 112.58 - (11^\circ - 11')\} \\
& + 3.405 \sin 2 \left\{ \begin{array}{cc} & \text{do} \end{array} \right\} \\
& + 283 \sin 3 \left\{ \begin{array}{cc} & \text{do} \end{array} \right\} \\
& = +20'8 \times \sin 144^\circ - 23' - 1.3783 \sin 199^\circ - 42' \\
& + 3.405 \sin 39^\circ - 24' + 283 \sin 239^\circ - 6' \\
& = +20'8 \times .5824 - 1.373 \times - .3371 + 3.405 \times .6347 \\
& + .283 \times - .8581 \\
& = +12'114 + 0'4628 + 2'1612 - 0'2428 = \underline{14' - 30''}
\end{aligned}$$

$\therefore$  Corrected mean longitude of Jupiter  $= 7^\circ - 26' - 17'' - 31''$

## POSITION OF APSE.

$$\begin{aligned}
\text{Mean motion per year} & = + 6''.63 \\
\text{Mean motion in 112.54 years} & = 12^\circ - 26' (+) \\
\text{H longitude of Apse at epoch} & = 5^\circ - 20' - 4'' - 37'' \\
\therefore \text{H longitude of Apse at birth} & = 5^\circ - 20' - 17'' - 3'' \\
\text{Mean anomaly} & = \left\{ \begin{array}{l} 5^\circ - 20' - 17'' - 3'' \text{ minus} \\ 7^\circ - 26' - 17'' - 31'' \end{array} \right. \\
& = \underline{9^\circ - 23' - 59'' - 32''} \\
& \quad 293^\circ - 59' - 32''
\end{aligned}$$

Equation of centre for Jupiter whose excentricity is .04833 is  
 $2 \sin nt (9966 - 599 \cos nt) + 26 \sin 3 nt$

In the present case,

$$\begin{aligned}
\sin nt & = \sin (293^\circ - 59' - 32'') \\
& = \sin (360^\circ - 66^\circ - 0' - 28'') \\
& = -\sin 66^\circ - 0' - 28'' = -.9136007 \\
\cos nt & = \cos (293^\circ - 59' - 32'') \\
& = \cos (360^\circ - 66^\circ - 0' - 28'') \\
& = \cos 66^\circ - 0' - 28'' = \sin 23^\circ - 59' - 32'' \\
& = .4066126 \\
\sin 3nt & = \sin (881^\circ - 58' - 36'') \\
& = \sin (720^\circ + 161^\circ - 58' - 36'') \\
& = \sin 161^\circ - 58' - 86'' \\
& = \sin 18^\circ - 1' - 24'' = .3094042
\end{aligned}$$

$$\begin{aligned}
\therefore \text{Equation of centre} & = 2 \times - .9136007 (9966 - 599 \times .4066) \\
& + 26 \times .3094
\end{aligned}$$

$$\begin{aligned}
 &= -17764'' + 8'' = -17756'' \\
 &= -295' - 56'' = -4^\circ - 55' - 56''
 \end{aligned}$$

Applying to the mean longitude we get

$$\begin{array}{l}
 \text{as the True Helo longitude } \left\{ \begin{array}{l} 7^\circ - 26' - 17'' - 31'' \\ \quad \quad \quad - 4^\circ - 55' - 56'' \\ \hline 7^\circ - 21' - 21'' - 35'' \end{array} \right.
 \end{array}$$

## TO OBTAIN THE RADIUS VECTOR.

$$\text{True longitude} = 7-21-21-35$$

$$\text{Apse} = 5-20-17-3$$

$$\text{True anomaly} = 9-28-55-28$$

$$= 298^\circ - 55' - 28''$$

$$\begin{aligned}
 \text{Radius vector} &= \frac{a(1-e^2)}{1-e \cos \theta} \text{ where } a \text{ for Jupiter is } 5.2028 \text{ and} \\
 e &= 0.4833
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Radius vector} &= \frac{5.19064}{1-e \cos \theta} \\
 &= \frac{5.19064}{1-0.4833 \cos 298^\circ - 55' - 28''} \\
 &= \frac{5.19064}{1-0.4833 \times .4847} = \frac{5.19064}{.976735} \\
 &= 5.3142
 \end{aligned}$$

From the tables equation of centre for argument  $66^\circ - 0' - 28''$  of the mean anomaly is found out thus —

for  $66^\circ$  equation of centre is  $4^\circ - 58' - 56''$

for  $67^\circ$  do is  $4^\circ - 58' - 27''$

$\therefore$  for  $66^\circ - 0' - 28''$  it is  $4^\circ - 55' - 56''$

The radius vector is directly read as 5.314 and the Hel Velocity as  $4'-46''$ . The Velocity is derived trigonometrically from the formula  $299-29 \cos nt + 2 \cos 2nt$ . As the mean velocity of Jupiter is not very great the results got from the tables will serve the purpose.



# TABLE OF EQUATION OF CENTRE OF JUPITER, HELIOCENTRIC VELOCITY AND RADIUS VECTOR.

Arg — Mean anomaly If greater than  $180^\circ$  then its defect from  $360^\circ$   
will be the argument but the equation of centre will be negative

De <sub>n</sub>	Equation of Centre			Radius Vector		Hel Velocity		Deg	Equation of Centre			Radius Vector		Hel Velocity	
	"			"		"			"			"		"	
0	0	0	0	5 453	4	32		35	3	1	34	5 411	4	36	
1	0	5	28	5 453	4	32		36	3	6	9	5 409	4	36	
2	0	10	56	5 452	4	32		37	3	10	42	5 406	4	36	
3	0	16	25	5 452	4	32		38	3	15	13	5 404	4	37	
4	0	21	52	5 452	4	32		39	3	19	40	5 401	4	37	
5	0	27	19	5 451	4	32		40	3	24	8	5 398	4	37	
6	0	32	46	5 451	4	32		41	3	28	24	5 396	4	38	
7	0	38	12	5 450	4	32		42	3	32	41	5 393	4	38	
8	0	43	38	5 450	4	32		43	3	36	55	5 390	4	38	
9	0	49	4	5 450	4	32		44	3	41	6	5 388	4	39	
10	0	54	29	5 449	4	32		45	3	45	12	5 385	4	39	
11	0	59	54	5 448	4	32		46	3	49	15	5 382	4	39	
12	1	5	15	5 448	4	32		47	3	53	15	5 379	4	40	
13	1	10	36	5 447	4	33		48	3	57	11	5 376	4	40	
14	1	15	58	5 446	4	33		49	4	1	2	5 373	4	41	
15	1	21	17	6 445	4	33		50	4	4	50	5 370	4	41	
16	1	26	35	5 444	4	33		51	4	8	34	5 367	4	41	
17	1	31	52	5 442	4	33		52	4	12	15	5 364	4	42	
18	1	37	8	5 441	4	33		53	4	15	51	5 361	4	42	
19	1	42	21	5 440	4	33		54	4	19	23	5 357	4	42	
20	1	47	33	5 438	4	33		55	4	22	50	5 354	4	42	
21	1	52	45	5 437	4	33		56	4	26	13	5 350	4	43	
22	1	57	53	5 436	4	34		57	4	29	32	5 347	4	43	
23	2	3	0	5 434	4	34		58	4	32	47	5 343	4	43	
24	2	8	5	5 433	4	34		59	4	35	56	5 340	4	44	
25	2	13	9	5 431	4	34		60	4	39	2	5 336	4	44	
26	2	18	10	5 430	4	34		61	4	42	3	5 332	4	45	
27	2	23	9	5 428	4	34		62	4	44	59	5 329	4	45	
28	2	28	5	5 426	4	35		63	4	47	50	5 325	4	45	
29	2	33	0	5 424	4	35		64	4	50	37	5 321	4	46	
30	2	37	52	5 422	4	35		65	4	53	18	5 318	4	46	
31	2	42	41	5 420	4	35		66	4	55	56	5 314	4	46	
32	2	47	28	5 418	4	36		67	4	58	27	5 310	4	47	
33	2	52	13	5 416	4	36		68	5	0	52	5 306	4	47	
34	2	56	54	5 414	4	36		69	5	3	15	5 302	4	48	

70	5	5	81	5 298	4	48	114	5	10	47	5 111	5	10
71	5	7	43	5 294	4	48	115	5	8	37	5 107	5	10
72	5	9	49	5 290	4	49	116	5	6	22	5 103	5	11
73	5	11	49	5 286	4	49	117	5	4	1	5 099	5	11
74	5	13	44	5 282	4	49	118	5	1	34	5 095	5	11
75	5	15	35	5 278	4	50	119	4	59	1	5 091	5	12
76	5	17	20	5 274	4	50	120	4	56	21	5 087	5	13
77	5	18	59	5 270	4	51	121	4	53	36	5 083	5	13
78	5	20	32	5 266	4	52	122	4	50	46	5 079	5	13
79	5	22	0	5 262	4	52	123	4	47	49	5 075	5	14
80	5	23	21	5 257	4	53	124	4	44	46	5 071	5	15
81	5	24	34	5 253	4	53	125	4	41	38	5 068	5	15
82	5	25	49	5 249	4	54	126	4	38	24	5 064	5	16
83	5	26	55	5 244	4	55	127	4	35	5	5 060	5	16
84	5	27	56	5 240	4	55	128	4	31	39	5 057	5	17
85	5	28	53	5 236	4	56	129	4	28	8	5 053	5	17
86	5	29	35	5 231	4	56	130	4	24	32	5 049	5	18
87	5	30	17	5 227	4	56	131	4	20	51	5 046	5	18
88	5	30	52	5 223	4	56	132	4	17	4	5 042	5	19
89	5	31	22	5 218	4	57	133	4	13	12	5 039	5	19
90	5	31	46	5 214	4	57	134	4	9	15	5 035	5	20
91	5	32	4	5 210	4	57	135	4	5	12	5 032	5	20
92	5	32	16	5 205	4	58	136	4	1	5	5 029	5	20
93	5	32	22	5 201	4	59	137	3	56	52	5 025	5	21
94	5	32	28	5 197	5	0	138	3	52	34	5 022	5	21
95	5	32	21	5 194	5	1	139	3	48	12	5 019	5	22
96	5	32	4	5 188	5	1	140	3	43	45	5 016	5	22
97	5	31	45	5 184	5	1	141	3	39	14	5 013	5	22
98	5	31	20	5 181	5	2	142	3	34	37	5 010	5	23
99	5	30	49	5 175	5	2	143	3	29	56	5 007	5	23
100	5	30	12	5 171	5	3	144	3	25	11	5 004	5	24
101	5	29	29	5 166	5	3	145	3	20	21	5 001	5	24
102	5	28	40	5 162	5	4	146	3	15	27	4 999	5	25
103	5	27	44	5 158	5	4	147	3	10	29	4 996	5	25
104	5	26	44	5 153	5	5	148	3	5	27	4 994	5	25
105	5	25	35	5 149	5	5	149	3	0	21	4 991	5	25
106	5	24	20	5 145	5	6	150	2	55	11	4 989	5	26
107	5	23	0	5 140	5	7	151	2	49	58	4 987	5	26
108	5	21	34	5 136	5	7	152	2	44	40	4 984	5	26
109	5	20	2	5 132	5	7	153	2	39	20	4 982	5	26
110	5	18	23	5 128	5	8	154	2	33	56	4 980	5	27
111	5	16	38	5 124	5	8	155	2	28	28	4 978	5	27
112	5	14	46	5 120	5	9	156	2	22	56	4 976	5	27
113	5	12	50	5 115	5	9	157	2	17	28	4 974	5	27

158	2	11	47	4 973	5	27	170	1	1	20	4 957	5	29
159	2	6	7	4 971	5	27	171	0	55	14	4 956	5	29
160	2	0	25	4 969	5	28	172	0	49	8	4 955	5	29
161	1	54	40	4 968	5	28	173	0	43	2	4 955	5	30
162	1	18	53	4 966	5	28	174	0	36	56	4 954	5	30
163	1	13	3	4 965	5	28	175	0	30	48	4 954	5	30
164	1	37	11	4 963	5	28	176	0	24	39	4 953	5	30
165	1	31	17	4 962	5	28	177	0	18	30	4 953	5	30
166	1	25	21	4 961	5	28	178	0	12	20	4 953	5	30
167	1	19	23	4 959	5	29	179	0	6	10	4 952	5	30
168	1	13	23	4 958	5	29	180	0	0	0	4 952	5	30
169	1	7	23	4 957	5	29							

### POSITION OF NODE

At epoch  $= 2^s-17^m-22^s-41^s$   
Annual motion of Node  $= -14^s 4$   
 $\therefore$  Motion in 112 54 years  $= -14^s 4 \times 112 54 = -27^m-1^s$   
 $\therefore$  Position at birth  $= 2^s-16^m-55^s-43^s$

### REDUCTION.

Heliocentric longitude of Jupiter  $7^s-21^m-21^s-35^s$   
Longitude of Node  $2^s-16^m-55^s-43^s$   
 $\therefore$  Nodal distance (NP)  $= 5^s-4^m-25^s-53^s$   
 $\therefore \tan NM = \cos \epsilon \tan NP$ , where  $\epsilon = 1^m-19^s$   
 $= \cos 1^m-19^s \tan 54^m-25^s-52^s$   
 $\therefore L \tan NM = L \cos 1^m-19^s + \tan 54^m-25^s-52^s = 10$   
 $= 9 9998853 + 9 6798386 = 10$   
 $= 9 6797239$   
 $= L \tan 54^m-26^s-13^s$   
 $\therefore NM = 54^m-26^s-13^s$   
 $\therefore$  True Heliocentric longitude of Jupiter along ecliptic  
 $= \begin{cases} 54^m-26^s-13^s & \text{plus} \\ 7^s-55^m-43^s \end{cases}$   
 $= 231^m-21^s-56^s = 7^s-21^m-21^s-56^s$

This can also be done by finding out the reduction from the Moon's reduction table with the same Nodal distance but applying the multiplier

$\frac{1}{16}$  to the reduction therefrom obtained The multiplied reduction should be applied to the orbital Hel Longitude of the planet previously obtained

## TO FIND LATITUDE.

$$\sin PM = \sin \iota \sin NP$$

$$= \sin 1^{\circ}-19' \sin 154^{\circ}-25'-52''$$

$$\therefore L \sin PM = L \sin 1^{\circ}-19' L \sin 154^{\circ}-25'-52'' = 10$$

$$= 8\ 3613150 + 9\ 6350774 = 10$$

$$= 7\ 9963924 = L \sin 34^{\circ}-6''$$

$$\therefore PM = 34^{\circ}-6'' \text{ This is } N \text{ or } (+e) \text{ as the Nodal distance is } < 180^{\circ}$$

$$SM = SP \cos \widehat{PSM} = 5\ 3142 \cos 34^{\circ}-6''$$

$$= 5.314$$

$$SE = 1\ 01646$$

$$\widehat{MSE} = (\widehat{TSM} - \widehat{TSE}) + 180^{\circ}$$

$$= (231^{\circ}-21'-56'') - (89^{\circ}-21'-22'') + 180^{\circ}$$

$$= 322^{\circ} - 0' - 34''$$

$$ME^2 = SM^2 + SE^2 - 2SE \cos \widehat{MSE}$$

$$= (5\ 314)^2 + (1\ 01646)^2 - 10\ 628 \times 1\ 01646 \times \cos (322^{\circ} - 0' - 34'')$$

$$= 28\ 2385 + 1\ 032 - 10\ 8028 \times 7880$$

$$= 28\ 2385 + 1\ 032 - 8\ 5060$$

$$= 20\ 7645 \therefore ME = \sqrt{20\ 7645} = 4\ 5568$$

$$\sin SME = \frac{SE}{ME} \sin \widehat{MSE} = \frac{1\ 01646}{4\ 5568} \sin 322^{\circ} - 0' - 34''$$

$$= \frac{1.01646}{4\ 5561} \times - \sin 37^{\circ}-53'-26''$$

$$= \frac{1\ 01646}{4\ 5568} \times - .6156485$$

$$= - .1373512$$

$$\therefore SME = - (7^{\circ}-53'-10'')$$

$$\therefore \angle \eta EM = \widehat{TSM} + \widehat{SME}$$

$$= \left\{ \begin{array}{l} 231^{\circ}-21'-56'' \\ 7^{\circ}-53'-40'' \end{array} \right.$$

$$= 223^{\circ}-28'-16'' = 7^{\circ}-13'-28'-16''$$

This is the geocentric longitude of Jupiter

## GEOCENTRIC VELOCITY.

This is = Velocity of  $\angle \text{'SM}$  + Velocity of  $\angle \text{SME}$

$$= (4' - 16'') + \frac{1\ 01646}{4\ 5568} \frac{\cos \text{MSE}}{\cos \text{SME}} \times \text{Rate of change } \angle \text{MSE}$$

$$= (4' - 46'') + \frac{1\ 01616}{4\ 5568} \times \frac{.7880}{9905} \times (-52' - 28'')$$

[because rate of change of  $\angle \text{MSE}$  = the difference of the Hel velocities of the planet and the earth]

$$\text{We get } (4' - 46'') - (9' - 18'') = -(4' - 32'')$$

Thus the planet is retrograde

## GEOCENTRIC LATITUDE.

$$\frac{PM}{SM} = \tan \angle \text{PSM} \text{ and } \frac{PM}{EM} = \tan \angle \text{PEM}$$

$$\therefore \tan \angle \text{PEM} = \frac{SM}{EM} \tan \angle \text{PSM}$$

$$= \frac{5\ 314}{4\ 5568} \times \tan 34' - 6''$$

$$= \frac{5\ 314}{4\ 5568} \times 0.0099196$$

$$= 0.0113487 = \tan 39' - 1''$$

$$\therefore \angle \text{PEM} = 39' - 1'' \text{ This also N or } + \text{ as the Hel Latitude is}$$

North

# Chapter XVI.

## VENUS.

### ELEMENTS

	s	°	'	''
1 Mean Heliocentric longitude at epoch	4	—	4	—58—30
2 Mean Hel longitude of Apse	9	—	17	—42—4
3 Mean Hel longitude of Node	1	—	23	—50—24

4	Length of semi major axis	•7233
5	Excentricity of orbit	•00681
6	Inclination of orbit to ecliptic	3°—23' 6
7	Periodic time	224•700 days
8	Annual motion of Apse	—1"52
9	Annual motion of Node	—19" 14
	No of days from epoch	=41101 82153
	Periodic time of Venus	=224 700 days

$$\text{No of revolutions} = \frac{41101\ 82153}{224\ 700} = 182\ 918645 \quad \text{Converting this}$$

fraction of a revolution we get 11<sup>s</sup>—0°—42'—44", to which if the epoch position 4<sup>s</sup>—4°—56'—30" be added we get 3<sup>s</sup>—5°—41'—14" as the mean longitude at birth

## HINDU METHOD.

Divide the number of days by 225 and leave off the quotient with the remainder get signs etc. Again divide by 471 and get degrees etc. These two when added give the mean motion for the required no. of days. The empirical correction for 1 000 000 one days is 782—22 additive

$$\frac{41101}{225} = 182 \frac{151}{225} = 8^s - 1^\circ - 36' - 0''$$

$$\frac{41101}{471} = 87^\circ \frac{124}{471} = 2 - 27 - 15 - 48$$

$$= 10 - 28 - 51 - 48$$

In 82153 of day at 96—8" per day 1—18—51

Empirical correction

$$= \frac{41102 \times 782 - 22}{1,000,000} = \underline{\underline{32' - 9''}}$$

Adding we get mean motion = 11<sup>s</sup>—0°—42'—51"

Epoch position = 4—1—58—30

$$\therefore \text{Mean position of Venus at birth} \left\{ \begin{array}{l} 3 - 5 - 41 - 21 \end{array} \right.$$

चरप्राणकलांतरदेशांतरादि is 28'—7"घने Therefore the correction due to this

is  $\frac{28' - 7" \times 96' - 8''}{21600} = 8''(+)$  Adding this we get Mean longitude of Venus at birth corrected to the observer is 3<sup>s</sup>—5°—41'—14" plus 8 = 3<sup>s</sup>—5°—41'—22

# TABLE OF MEAN MOTION OF VENUS.

Periodic of revolution = 224 700 days

Day	°	Degrees	Minutes	Seconds	Days	°	Degrees	Minutes	Seconds	Days	°	Degrees	Minutes	Seconds
1	0	1	36	8	300	1	0	38	27	50000	6	6	48	33
2	0	3	12	15	400	9	10	51	16	60000	0	8	10	15
3	0	4	48	23	500	2	21	4	5	70000	6	9	31	58
4	0	6	24	31	600	8	1	16	54	80000	0	10	53	40
5	0	8	0	39	700	1	11	29	43	90000	6	12	15	23
6	0	9	36	46	800	6	21	42	32	100000	0	13	37	5
7	0	11	12	54	900	0	1	55	21	200000	0	27	14	11
8	0	12	49	2	1000	5	12	8	10	300000	1	10	51	16
9	0	14	25	14	2000	10	24	16	21	400000	1	24	28	22
10	0	16	1	17	3000	4	6	24	31	500000	2	8	5	27
20	1	2	2	34	4000	9	18	32	41	600000	2	21	42	32
30	1	18	3	51	5000	3	0	40	52	700000	3	5	19	38
40	2	4	5	8	6000	8	12	49	2	800000	3	18	56	43
50	2	20	6	25	7000	1	24	57	12	900000	4	2	33	49
60	3	6	7	41	8000	7	7	5	22	1000000	4	16	10	54
70	3	22	8	58	9000	0	19	13	33					
80	4	8	10	15	10000	6	1	21	13					
90	4	24	11	32	20000	0	2	43	25					
100	5	10	12	49	30000	6	4	5	8					
200	10	20	25	38	40000	0	5	26	50					

## FROM THE TABLES

	°	'	"
40000 days	0	--5	--26--50
1000 days	5	--12	--8--10
100 days	5	--10	--12--49
1 day	0	--1	--36--8
	10	--29	--23--57
82153 day	0	--1	--18--54
	11	--0	--42--51
Epoch	4	--4	--58--30
	3	--5	--41--21

Correction	+8
Mean longitude of Venus at birth	<u>3—5—41—29</u>

This is very nearly that of the other method

## POSITION OF APSE.

	At epoch	9°—17'—42"—4"
Motion in 11254 years @ — 1" 52 per year =		<u>2—51"</u>
∴ Position of Apse at birth		<u>= 9°—17'—39"—13"</u>
Mean anomaly	= {	9—17—39—13 minus 3—5—41—22
		<u>= 6—11—57—51</u>
		or 191°—57'—51"

Equation of centre for Venus whose excentricity is 00681 in given by the formula  $\sin nt$  (2809—24  $\cos nt$ ) in seconds of arc

In the present case  $nt = 191^{\circ}-57'-51''$  and

$$\begin{aligned}
 \therefore \text{Equation of centre} &= \sin 191^{\circ}-57'-51'' (2809-24 \cos 191^{\circ}-57'-51'') \\
 &= -\sin 11^{\circ}-57'-51'' (2809+24 \cos 11^{\circ}-57'-51'') \\
 &= -2073 (2809+24 \times .9782774) \\
 &= -2073 (2809+23.4785) \\
 &= -2073 \times 2832.4785 \\
 &= -587'' = -9'-47''
 \end{aligned}$$

Applying this to the mean longitude of Venus we get 3°—5'—41"—22" minus 9'—47" of 3°—5'—31'—35"

## RADIUS VECTOR.

$$r = \frac{a(1-e^2)}{1-e \cos \theta}, \text{ where } a = .7233 \text{ and } e = .00681 \text{ for venus}$$

It reduces to  $\frac{.72327}{1-00681 \cos \theta}$  where  $\theta$  is the true anomaly

$$\begin{aligned}
 \text{In the present case True anomaly} &= \begin{cases} 9^{\circ}-17'-39'-13'' \\ 3^{\circ}-5'-31'-35'' \end{cases} \\
 &= \begin{cases} 6^{\circ}-12'-7'-38'' \\ 192^{\circ}-7'-38'' \end{cases}
 \end{aligned}$$



$$\begin{aligned}\therefore r &= \frac{.72327}{1 - 00681 \cos 192^\circ - 7' - 38''} \\ &= \frac{72327}{1 + 00681 \times 9777} = \frac{72327}{1 + 0065} = \frac{.72327}{1.0065} \\ &= \underline{7186}\end{aligned}$$

## HELIOCENTRIC VELOCITY.

This is given by the formulae in mts of arc as  $(96' 13 - 1' 31 \cos nt)$

In the present case  $\cos nt = \cos 191^\circ - 57' - 51''$

$$= -\cos 11^\circ - 57' - 51'' = -.9788$$

$$\therefore \text{Vel} = 96' 13 + 1' 28 = 97' 41 = 97 - 25$$

We shall now arrive at all these things from the tables appended below

## TABLE OF EQUATION OF CENTRE, RADIUS VECTOR AND HELIOCENTRIC VELOCITY OF VENUS.

Arg — Mean anomaly If more than  $180^\circ$  the defect from  $360^\circ$  will be the argument to refer to the table but the equation of centre will be then negative

Deg	Equation of Centre			Radius Vector	Hel Velocity			Deg	Equation of Centre			Radius Vector	Hel Velocity		
°	'	0	0					°	'	0	0				
0	0	0	0	7283	94	50		16	0	12	48	7281	94	53	
1	0	0	49	7283	94	50		17	0	18	35	7281	94	54	
2	0	1	37	7283	94	50		18	0	14	21	7281	94	54	
3	0	2	26	7283	94	50		19	0	15	7	7281	94	55	
4	0	3	16	7283	94	50		20	0	15	33	7280	94	55	
5	0	4	3	7283	94	51		21	0	16	40	7780	94	56	
6	0	4	51	7283	94	51		22	0	17	24	7280	94	56	
7	0	5	39	7283	94	52		23	0	18	9	7279	94	57	
8	0	6	28	7283	94	52		24	0	18	54	7279	94	58	
9	0	7	16	7283	94	52		25	0	19	38	7279	94	58	
10	0	8	4	7283	94	52		26	0	20	22	7278	94	59	
11	0	8	51	7282	94	52		27	0	21	6	7278	94	59	
12	0	9	39	7282	94	52		28	0	21	49	7278	95	0	
13	0	10	27	7282	94	53		29	0	22	32	7277	95	0	
14	0	11	14	7282	94	53		30	0	23	14	7276	95	1	
15	0	12	1	7282	94	53		31	0	23	56	7276	95	1	

32	0 24 38	.7275	95	2	76	0 45 21	.7246	95	48
33	0 25 19	.7275	95	2	77	0 45 32	.7245	95	50
34	0 26 0	.7275	95	3	78	0 45 43	.7244	95	52
35	0 26 40	.7274	95	4	79	0 45 53	.7243	95	54
36	0 27 20	.7274	95	6	80	0 46 2	.7242	95	55
37	0 27 59	.7274	95	6	81	0 46 11	.7242	95	57
38	0 28 38	.7273	95	7	82	0 46 18	.7241	95	58
39	0 29 16	.7273	95	7	83	0 46 25	.7240	95	59
40	0 29 54	.7273	95	8	84	0 46 31	.7239	96	0
41	0 30 31	.7272	95	9	85	0 46 36	.7238	96	1
42	0 31 8	.7271	95	10	86	0 46 40	.7237	96	2
43	0 31 44	.7271	95	10	87	0 46 44	.7237	96	3
44	0 32 19	.7270	95	11	88	0 46 47	.7236	96	5
45	0 32 54	.7269	95	12	89	0 46 48	.7235	96	6
46	0 33 29	.7268	95	14	90	0 46 49	.7234	96	8
47	0 34 2	.7268	95	15	91	0 46 49	.7233	96	9
48	0 34 36	.7267	95	17	92	0 46 48	.7232	96	11
49	0 35 8	.7266	95	17	93	0 46 47	.7231	96	13
50	0 35 40	.7265	95	18	94	0 46 44	.7230	96	14
51	0 36 11	.7265	95	18	95	0 46 40	.7229	96	15
52	0 36 42	.7264	95	19	96	0 46 36	.7229	96	16
53	0 37 12	.7263	95	20	97	0 46 31	.7228	96	17
54	0 37 41	.7263	95	22	98	0 46 25	.7227	96	18
55	0 38 10	.7262	95	23	99	0 46 18	.7226	96	19
56	0 38 38	.7262	95	25	100	0 46 11	.7225	96	21
57	0 39 5	.7261	95	26	101	0 46 2	.7224	96	24
58	0 39 31	.7261	95	27	102	0 45 53	.7224	96	26
59	0 39 57	.7260	95	28	103	0 45 42	.7223	96	26
60	0 40 22	.7259	95	28	104	0 45 31	.7222	96	27
61	0 40 47	.7258	95	29	105	0 45 19	.7221	96	27
62	0 41 10	.7257	95	30	106	0 45 7	.7220	96	29
63	0 41 33	.7256	95	31	107	0 44 53	.7219	96	30
64	0 41 56	.7255	95	33	108	0 44 39	.7218	96	32
65	0 42 17	.7254	95	34	109	0 44 24	.7217	96	33
66	0 42 37	.7254	95	36	110	0 44 7	.7216	96	34
67	0 42 57	.7253	95	37	111	0 43 50	.7216	96	35
68	0 43 16	.7252	95	38	112	0 43 33	.7215	96	37
69	0 43 34	.7252	95	39	113	0 43 14	.7214	96	38
70	0 43 52	.7251	95	41	114	0 42 55	.7214	96	40
71	0 44 9	.7250	95	43	115	0 42 35	.7213	96	42
72	0 44 25	.7249	95	44	116	0 42 14	.7212	96	43
73	0 44 40	.7249	95	45	117	0 41 53	.7211	96	45
74	0 44 54	.7248	95	46	118	0 41 30	.7210	96	45
75	0 45 7	.7247	95	46	119	0 41 7	.7210	96	46

120	0	40	48	-7209	96	46	151	0	22	52	7190	97	18
121	0	40	19	7208	96	48	152	0	22	9	7189	97	18
122	0	39	53	7207	96	49	153	0	21	25	7189	97	18
123	0	39	27	7206	96	51	154	0	20	41	7189	97	19
124	0	39	0	7205	96	52	155	0	19	56	7188	97	20
125	0	38	32	7204	96	53	156	0	19	11	7188	97	20
126	0	38	4	7204	96	54	157	0	18	26	7187	97	20
127	0	37	35	-7203	96	55	158	0	17	41	7187	97	21
128	0	37	5	7203	96	57	159	0	16	56	7187	97	22
129	0	36	35	7202	96	58	160	0	16	8	7187	97	22
130	0	36	4	7201	96	59	161	0	15	22	8186	97	23
131	0	35	32	-7201	97	0	162	0	14	35	7186	97	23
132	0	35	0	7200	97	1	163	0	13	48	7186	97	24
133	0	34	27	7199	97	2	164	0	13	1	7186	97	25
134	0	33	53	7198	97	3	165	0	12	13	7185	97	25
135	0	33	18	7198	97	4	166	0	11	25	7185	97	26
136	0	32	43	-7198	97	5	167	0	10	37	7185	97	26
137	0	32	8	7197	97	6	168	0	9	49	7185	97	26
138	0	31	32	7197	97	7	169	0	9	0	7185	97	26
139	0	30	55	7196	97	8	170	0	8	12	7184	97	26
140	0	30	17	7196	97	9	171	0	7	23	7184	97	26
141	0	29	40	-7195	97	10	172	0	6	34	7184	97	26
142	0	29	1	7194	97	11	173	0	5	45	-7184	97	26
143	0	28	22	7194	97	13	174	0	4	56	7184	97	26
144	0	27	43	7193	97	14	175	0	4	7	7184	97	26
145	0	27	2	7192	97	14	176	0	3	19	7183	97	26
146	0	26	22	7192	97	15	177	0	2	28	7183	97	26
147	0	25	41	7192	97	15	178	0	1	39	7183	97	26
148	0	24	59	7191	97	16	179	0	0	49	7183	97	27
149	0	24	17	-7191	97	17	180	0	0	0	7183	97	27
150	0	23	35	7191	97	17							

## FROM THE TABLES.

Mean anomaly is  $6^{\circ}-11^{\circ}-57'-51''$   
or  $191^{\circ}-57'-51''$

As this is more than  $180^{\circ}$  defect from  $360^{\circ}$  is  $168^{\circ}-2'-9''$ , this will be the argument to refer to the tables

Equation of centre for  $168^{\circ}$  is  $0^{\circ}-9'-49''$  and difference for  $1^{\circ}$  is  $0^{\circ}-0'-49''$  decreasing

\*. Correct equation of centre for  $168^{\circ}-2'-3''$  is  $0^{\circ}-9'-47''$  As the mean anomaly is greater than  $180^{\circ}$ , the equation of centre is negative

∴ Subtracting it from the mean longitude we get  $3^{\circ}-5'-31''-35''$  as the true longitude. Against the same mean anomaly, the radius vector and Heliocentric Velocity are read as 7185 and  $97-26''$

### POSITION OF NODE

At epoch	= $1^{\circ}-23'-50''-24''$
Annual motion	= $-19'' 14$
∴ Motion in 11254 years	= $-35^{\circ} 54$
∴ Position of Node at birth	= $1^{\circ}-23'-14''-30''$

### REDUCTION.

True longitude of Venus ( $\Upsilon^{\circ}P$ )	= $3^{\circ}-5'-31''-35''$
Longitude of Node ( $\Upsilon^{\circ}N$ )	= $1^{\circ}-23'-14''-30''$
∴ Nodal distance (NP)	= $1^{\circ}-12'-17''-5''$
	or $42^{\circ}-17'-5''$

∴  $\tan NM = \cos \iota \tan NP$ , where  $\iota$  in this case of Venus' orbit is  $3^{\circ}-23' 6$

$$\begin{aligned}\therefore L \tan NM &= L \cos 3^{\circ}-23' 6 + L \tan 42^{\circ}-17'-5''-10 \\ &= 99992409 + 99587754 - 10 \\ &= 99580163 = L \tan 42^{\circ}-14'-6''\end{aligned}$$

$$\begin{aligned}\therefore \text{True Heliocentric longitude of Venus referred to the ecliptic is} \\ &42^{\circ}-14'-6'' \text{ plus} \\ &58-14-30 \\ &= 95-28-36 \\ &= 3^{\circ}-5'-28''-36''\end{aligned}$$

The reduction can also be got by using the multiplier  $\frac{1}{6}$  to the reduction got from the Moon's tables with the Nodal distance of the planet as the argument

### HELIOCENTRIC LATITUDE (P M)

$$\begin{aligned}\sin PM &= \sin \iota \sin NP \\ &= \sin 3^{\circ}-23' 6 \times \sin 42^{\circ}-17'-5'' \\ \therefore L \sin PM &= L \sin 3^{\circ}-23' 6 + L \sin 42^{\circ}-17'-5 - 10 \\ &= 87722467 + 98360862 - 10 \\ &= 86083349 \\ &= L \sin 2^{\circ}-19'\end{aligned}$$

$\therefore PM = 2^{\circ}-19' (N)$ . This is North or positive as the Nodal distance is less than  $180^{\circ}$ .

$$SM = SP \cos \hat{PSM} = 7186 \cos 2^{\circ}-19'$$

$$= 7186 \times 9991827 = 7180$$

$$SE = 101646$$

$$\angle MSE = \angle \uparrow SM - \angle \uparrow SE + 180^{\circ}$$

$$= (95^{\circ}-28'-36'') - (89^{\circ}-21'-22'') + 180^{\circ}$$

$$= 186^{\circ}-7'-14''$$

$$\therefore ME^2 = SM^2 + SE^2 - 2SM SE \cos \hat{MS E}$$

$$= (718)^2 + (101646)^2 - 2 \times 718 \times 101646 \times \cos 186^{\circ}-7'-14''$$

$$(\cos 186^{\circ}-7'-14'' = -\cos 6^{\circ}-7'-14'' = -.9943)$$

$$\therefore ME^2 = 5155 + 1032 + 14597 \times 9943$$

$$= 15475 + 14514 = 29989 \quad \therefore ME = \sqrt{29989} = 17432$$

$$\frac{ME}{\sin \angle MSE} = \frac{SE}{\sin \angle SME} \quad \therefore \sin \angle SME = \frac{SE}{ME} \sin \angle MSE$$

$$(ie) \sin \angle SME = \frac{101646}{17432} \sin 186^{\circ}-7'-14''$$

$$= \frac{101646}{17432} \times -\sin 6^{\circ}-7'-14''$$

$$= -\frac{101646 \times .1066208}{17432}$$

$$= -.621706$$

$$= \sin (-3^{\circ}-33'-52'')$$

$$\therefore \angle SME = -(3^{\circ}-33'-52'')$$

In the  $\triangle SME$ ,  $\angle MSE = 186^{\circ}-7'-14''$  this being more than  $180^{\circ}$ ; take the defect from 360 it is  $173^{\circ}-52'-46''$ ,  $\angle SME = 3^{\circ}-33'-52''$ .

$$\therefore \angle SEM = 180^{\circ} - (173^{\circ}-52'-46'' + 3^{\circ}-33'-52'')$$

$$= 180^{\circ} - (177^{\circ}-26'-38'') = 2^{\circ}-33'-22''$$

Venus being an inferior planet the  $\angle SME$  should be numerically always greater than  $\angle SEM$  (see notes under Mercury) It is found to be satisfied in this case taking the numerical values only into consideration.

$$\therefore \text{Geocentric longitude} = \uparrow SM + \hat{SME}$$

$$= \frac{95^{\circ}-28' \quad 36''}{3^{\circ}-33'-52''}$$

$$= \frac{91^{\circ}-54'-44''}{\phantom{00}} = 3^{\circ}-1^{\circ}-54'-42''$$

## GEOCENTRIC VELOCITY.

$$\begin{aligned}
 \text{This is} &= \text{Velocity of } \angle TSM + \text{Velocity of } \angle SME \\
 &= \text{Rate of change of } \angle TSM + \text{rate of change of } \angle SME \\
 &= \text{Heliocentric Velocity} + \text{rate of change of } \angle SME
 \end{aligned}$$

Rate of change of  $\angle SME$

$$\begin{aligned}
 &= \left[ \frac{SE}{ME} \frac{\cos \hat{MSE}}{\cos \hat{SME}} - \frac{SM}{ME} \frac{SE}{ME} \times \frac{\sin^2 \hat{MSE}}{\cos \hat{SME}} \right] \times \text{Diff of Hel velocities of Venus and Sun} \\
 &= \left[ \frac{1.01616}{1.7482} \frac{\cos 186^\circ - 7' - 11''}{\cos (-3^\circ - 33' - 52'')} - \frac{.718 \times 1.01616}{(1.7482)^2} \frac{\sin^2 186^\circ - 7' - 11''}{\cos (-3^\circ - 33' - 52'')} \right] \\
 &\quad \times (97' - 25'' - 57' - 14'') \\
 &= \left[ \frac{1.01616 \times -.9913}{1.7482 \times .9980} - \frac{.718 \times 1.01616 \times (-.1066)^2}{(1.7482)^2 \times .9980} \right] \times (40' - 11'') \\
 &= [-.5819 - .00159] \times 40' - 18'' \\
 &= -.58349 \times 40' - 18'' = -(23' - 28'') \\
 \therefore \text{Geocentric Velocity} &= (97' - 25'') - (23' - 28'') \\
 &= \underline{\underline{(73' - 57'')}}
 \end{aligned}$$

## GEOCENTRIC LATITUDE.

$$\frac{PM}{SM} = \tan \angle PSM \text{ and } \frac{PM}{EM} = \tan \angle PEM$$

$$\begin{aligned}
 \therefore \tan \angle PEM &= \frac{SM}{EM} \tan \angle PSM \\
 &= \frac{.718}{1.748} \tan 2^\circ - 19' \text{ (N)} \\
 &= \frac{.718}{1.748} \times .0404555 = 0.16665 \\
 \therefore \angle PEM &= 0^\circ - 57' - 14'' \text{ (N)}
 \end{aligned}$$

This is the geocentric latitude of planet



# Chapter XVII.

## \* SATURN.

### ELEMENTS

1	Mean Heliocentric longitude at epoch	3—12—2—23
2	Mean Hel longitude of Apse at epoch	8—8—6—40
3	Mean Hel longitude of Node at epoch	3—0—58—16
4	Length of semi major axis	9 5547
5	Excentricity of orbit	05589
6	Inclination of orbit to the ecliptic	2°—49 9
7	Periodic time	10759 22 days
8	Annual motion of Apse	= +16 10
9	Annual motion of Node	= —18 57
	No of days from epoch	~41101 82153
	Periodic time	= 10759 22 days
	*. No of revolutions =	$\frac{41101\ 82153}{10759\ 22}$
		= 3 820149

Leaving off the no of exact revolutions and converting the decimal portion to signs etc we get 9°—25°—15—13 If to this epoch mean position 3°—12°—2—23" be added we get mean longitude at birth as 1°—7°—17—36

### HINDU METHOD.

Divide the number of days by 900 we get signs etc Again divide by 146 the number of days The quotient will be minutes etc The sum of the two will be the mean longitude Empirical correction is 731—9 for every 1 000 000 days additive

In the example

$$\frac{41101}{900} = 45 \quad \frac{601}{900} = 45^{\circ}-20^{\circ}-2-0''$$

$$\frac{41101}{146} = 291' \quad \frac{124}{146} = \underline{4-41-31}$$

$$= 45-24-43-31$$

$$= 9-24-43-31$$

$$\text{Motion in } 82153 \text{ of a day} = \underline{1-38}$$

$$\text{.Adding we get} = \underline{9^{\circ}-24^{\circ}-45'-0''}$$

$$\text{Empirical correction} = \frac{731' \cdot 15 \times 11102}{1,000,000} = 30' \ 3''$$

$$\therefore \text{Correct mean longitude of saturn} = 9^s - 25' - 15'' - 12'' \text{ plus}$$

$$\begin{array}{r} 3 - 12 - 2 - 23 \\ \hline 1 - 37 - 17 - 35 \end{array} \text{ at epoch}$$

$$\text{Effect of चरप्राणकलांतरदेशांतरबाहुफलं is nil, because } \frac{26' - 7'' \times 2'}{21600} = \text{nil}$$

$\therefore$  The mean longitude already obtained is sufficient, it is  $1^s - 7' - 17'' - 36''$

## TABLE OF MEAN MOTION OF SATURN.

Periodic time = 10759 22 days

Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds
1	0	0	2	0	300	0	10	2	17	50000	7	22	59	2
2	0	0	4	1	400	0	13	23	2	60000	6	27	31	50
3	0	0	6	1	500	0	16	43	18	70000	6	26	10	38
4	0	0	8	2	600	0	20	4	33	80000	5	6	46	26
5	0	0	10	2	700	0	23	25	19	90000	4	11	22	15
6	0	0	12	3	800	0	26	46	4	100000	3	15	58	3
7	0	0	14	3	900	1	0	6	49	200000	7	1	56	6
8	0	0	16	4	1000	1	3	27	35	300000	10	17	54	8
9	0	0	18	4	2000	2	6	55	10	400000	2	3	52	11
10	0	0	20	5	3000	3	10	22	41	500000	5	19	50	14
20	0	0	40	9	4000	4	18	50	19	600000	9	5	48	17
30	0	1	0	14	5000	5	17	17	54	700000	0	21	46	20
40	0	1	20	18	6000	6	20	15	29	800000	4	7	44	22
50	0	1	40	23	7000	7	24	13	4	900000	7	23	42	25
60	0	2	0	28	8000	8	27	40	38	1000000	11	9	40	28
70	0	2	20	32	9000	10	1	8	13					
80	0	2	40	37	10000	11	4	35	48					
90	0	3	0	41	20000	10	9	11	37					
100	0	3	40	46	30000	9	13	47	25					
200	0	6	41	31	40000	8	18	23	13					

### FROM THE TABLES

40000 days

8-18-23-13

1000 days

1-3-27-35



100 days	0—3—20—46
1 day	0—0—2—0
82153 day	0—0—1—38
Epoch	3—12—2—23
Total	<u>1—7—17—35</u>

## CORRECTION TO MEAN SATURN DUE TO ATTRACTION BETWEEN JUPITER AND SATURN.

Saturn's correction

$$\begin{aligned}
 &= -48^{\circ} 7' \sin (t \times .4074926) \\
 &+ 7' \sin (163^{\circ} 48' - 5^{\circ} 8945 \times t) \\
 &+ 10^{\circ} 85' \sin (243^{\circ} 15' - 11^{\circ} 794 t)
 \end{aligned}$$

where  $t$  is the number of year's from 1558 AD March.

In the present case, the year of birth is 1912 July and therefore,  
 $t = 1912 \text{ July} - 1558 \text{ March} = 354.33$

$$\begin{aligned}
 t \times .4074926 &= 354.33 \times .4074926 = 144^{\circ} 38' \\
 163^{\circ} 48' - 5.8945 \times 354.33 &= (163^{\circ} - 29') - (2088^{\circ} - 16') \\
 &= -(1925^{\circ} - 7') = -(125^{\circ} - 7') \\
 243^{\circ} 15' - 11.794 \times 354.33 &= (243^{\circ} - 9') - (4178^{\circ} - 58') \\
 &= -(3935^{\circ} - 49') = (335^{\circ} - 49') \\
 &= 24^{\circ} - 11'
 \end{aligned}$$

$\therefore$  Saturn's correction

$$\begin{aligned}
 &= -48^{\circ} 7' \sin 144^{\circ} 38' + 7' \sin (-(125^{\circ} - 7')) + 10^{\circ} 85' \sin (24^{\circ} - 11') \\
 &= -48^{\circ} 7' \times .5824 + 7' \times (-.818) + 10^{\circ} 85' \times .4097 \\
 &= -28^{\circ} 35' - 5^{\circ} 726 + 4^{\circ} 444 = -29^{\circ} 632 = -29^{\circ} - 38'
 \end{aligned}$$

$\therefore$  Mean longitude of Saturn already arrived at is  $1^{\circ} - 7' - 17' - 36''$ ;

applying the attraction correction, we get  $1^{\circ} - 6^{\circ} - 47' - 58''$

## POSITION OF APSE.

At epoch is	8 <sup>s</sup> —8°—6'—40"
Annual motion of apse	= +16'.10
$\therefore$ Motion in 112.54 years	= +16' 10" $\times$ 112.54 = 30'—12"
$\therefore$ H. longitude of Apse at required time	$\left\{ \begin{array}{l} \text{8—8—36—52} \end{array} \right.$
Mean anomaly	$= \left\{ \begin{array}{l} 8-8-36-52 \text{ minus} \\ 1-6-47-58 \\ \hline 7-1-48-54 = 211^{\circ} - 48' - 54'' \end{array} \right.$

## EQUATION OF CENTRE.

The formula in the case of Saturn whose excentricity of the orbit is 05589, is in seconds of arc

$$23047 \sin nt - 805 \sin 2 nt + 39 \sin 3 nt$$

In the present instance

$$\begin{aligned} \sin nt &= \sin (211^\circ - 48' - 54'') = \sin (180^\circ + 31^\circ - 48' - 54'') \\ &= - \sin (31^\circ - 48' - 54'') = - 5271783 \end{aligned}$$

$$\begin{aligned} \sin 2 nt &= \sin (423^\circ - 37' - 18'') = \sin (360^\circ + 63^\circ - 37' - 18'') \\ &= \sin 63^\circ - 37' - 18'' = 8959445 \end{aligned}$$

$$\begin{aligned} \sin 3 nt &= \sin (635^\circ - 26' - 42'') = \sin (360^\circ + 275^\circ - 26' - 42'') \\ &= \sin (360^\circ - 84^\circ - 33' - 18'') \\ &= - \sin 84^\circ - 33' - 18'' = - 9954878 \end{aligned}$$

$$\begin{aligned} \text{* Equation of centre} &= 23047 \times - 5271783 - 805 \times 8959445 \\ &\quad + 39 \times - 9954878 \\ &= - 121498 - 7212 - 388 \\ &= - 12909'' 8 = - (3^\circ - 35' - 10'') \end{aligned}$$

Applying this to the mean longitude we get  $1^s - 6^\circ - 47' - 58''$  minus  $3^\circ - 35' - 10''$  or  $1^s - 3^\circ - 12' - 48''$  as the true longitude

## RADIUS VECTOR.

This is =  $\frac{a(1-e^2)}{1-e \cos \theta}$  where  $a = 9.5517$   $e = 0.5589$  and  $\theta =$  the true anomaly

$$\begin{aligned} \therefore \text{Radius vector} &= \frac{9.5547 \times (1 - 0.5559^2)}{1 - 0.5589 \cos \theta} \\ &= \frac{9.5246}{1 - 0.5589 \cos \theta} \end{aligned}$$

In the present case

$$\text{True anomaly } (\theta) = \begin{cases} 8^\circ - 8' - 36'' - 52'' - \\ 1^\circ - 3^\circ - 12' - 48'' \\ \hline 7^\circ - 5^\circ - 24' - 4'' = 215^\circ - 24' - 4'' \end{cases}$$

$$\begin{aligned} \text{* Radius vector} &= \frac{9.5246}{1 - 0.5589 \times \cos 215^\circ - 24' - 4''} \\ &= \frac{9.5246}{1 + 0.5589 \cos (35^\circ - 24' - 4'')} = \frac{9.5246}{1 + 0.5589 \times 81512} \\ &= \frac{9.5246}{1.04556} = \frac{9.1096}{1.04556} \end{aligned}$$

# HELIOCENTRIC VELOCITY.

This is found from the formula in seconds of arc as  $(120-13 \cos nt + \cos 2 nt)$

The following table will give the equation of centre radius vector and Hel velocity for each degree of mean anomaly —

## TABLE OF EQUATION OF CENTRE, RADIUS VECTOR AND HELIOCENTRIC VELOCITY OF SATURN

Arg — Mean anomaly If greater than  $180^\circ$  its defect from  $360^\circ$  will be the argument but the equation of centre will be negative

Deg	Equation of Centre "			Radius Vector	Hel Velocity	Deg	Equation of Centre "			Radius Vector	Hel Velocity
0	0	0	0	10 082	1 47	27	2	44	11	10 080	1 49
1	0	6	16	10 082	1 47	28	2	49	51	10 026	1 49
2	0	12	32	10 082	1 47	29	2	55	30	10 022	1 49
3	0	18	48	10 082	1 47	30	3	1	6	10 017	1 49
4	0	25	3	10 081	1 48	31	3	6	38	10 013	1 49
5	0	31	19	10 081	1 48	32	3	12	8	10 008	1 49
6	0	37	34	10 080	1 48	33	3	17	35	10 004	1 49
7	0	43	48	10 079	1 48	34	3	23	0	9 999	1 50
8	0	50	2	10 077	1 48	35	3	29	20	9 994	1 50
9	0	56	14	10 076	1 48	36	3	33	39	9 990	1 50
10	1	2	26	10 075	1 48	37	3	38	52	9 985	1 50
11	1	8	37	10 074	1 48	38	3	44	5	9 979	1 50
12	1	14	47	10 072	1 48	39	3	49	11	9 974	1 50
13	1	20	57	10 070	1 48	40	3	54	16	9 968	1 50
14	1	27	4	10 068	1 48	41	3	59	15	9 963	1 50
15	1	33	10	10 066	1 48	42	4	4	13	9 957	1 50
16	1	39	15	10 064	1 48	43	4	9	6	9 951	1 50
17	1	45	15	10 062	1 48	44	4	13	55	9 945	1 50
18	1	51	21	10 059	1 48	45	4	18	40	9 939	1 50
19	1	57	20	10 056	1 48	46	4	23	16	9 933	1 50
20	2	3	19	10 053	1 48	47	4	27	56	9 927	1 51
21	2	9	15	10 050	1 48	48	4	32	29	9 921	1 51
22	2	15	10	10 047	1 48	49	4	36	52	9 914	1 51
23	2	21	2	10 044	1 48	50	4	41	21	9 908	1 51
24	2	26	52	10 041	1 48	51	4	45	42	9 901	1 52
25	2	32	41	10 037	1 48	52	4	49	56	9 894	1 52
26	2	38	27	10 034	1 49	53	4	54	6	9 887	1 52

54	4	58	12	9 880	1	52	98	6	28	28	9 501	2	1
55	5	2	13	9 873	1	52	99	6	22	56	9 491	2	1
56	5	6	9	9 865	1	52	100	6	22	17	9 482	2	1
57	5	9	59	9 858	1	52	101	6	21	31	9 472	2	2
58	5	13	45	9 851	1	53	102	6	20	38	9 468	2	2
59	5	17	27	9 843	1	53	103	6	19	37	9 454	2	2
60	5	21	2	9 836	1	53	104	6	18	30	9 444	2	2
61	5	24	33	9 828	1	53	105	6	17	15	9 435	2	3
62	5	27	58	9 820	1	53	106	6	15	53	9 426	2	3
63	5	31	17	9 812	1	53	107	6	14	24	9 417	2	3
64	5	34	32	8 804	1	54	108	6	12	48	9 408	2	3
65	5	37	40	9 796	1	54	109	6	11	4	9 399	2	3
66	5	40	44	9 788	1	54	110	6	9	13	9 389	2	4
67	5	43	41	9 780	1	54	111	6	7	14	9 380	2	4
68	5	46	33	9 772	1	54	112	6	5	10	9 371	2	4
69	5	49	18	8 764	1	54	113	6	2	58	9 362	2	5
70	5	52	0	9 755	1	55	114	6	0	39	9 353	2	5
71	5	54	34	9 747	1	55	115	5	58	12	9 344	2	5
72	5	57	3	9 738	1	55	116	5	55	39	9 336	2	5
73	5	59	24	9 729	1	56	117	5	52	58	9 327	2	6
74	6	1	41	9 721	1	56	118	5	50	11	9 318	2	6
75	6	3	51	9 712	1	56	119	5	47	17	9 310	2	6
76	6	5	55	9 702	1	56	120	5	44	15	9 301	2	6
77	6	7	52	9 693	1	56	121	5	41	6	9 292	2	6
78	6	9	44	9 683	1	56	122	5	37	50	9 284	2	7
79	6	11	29	9 675	1	56	123	5	34	28	9 275	2	7
80	6	13	7	9 667	1	57	124	5	31	0	9 267	2	7
81	6	14	39	9 659	1	57	125	5	27	24	9 259	2	7
82	6	16	4	9 650	1	57	126	5	23	42	9 251	2	8
83	6	17	24	9 641	1	57	127	5	19	52	9 243	2	8
84	6	18	36	9 632	1	58	128	5	15	56	9 235	2	8
85	6	19	40	9 623	1	58	129	5	11	55	9 227	2	8
86	6	20	39	9 613	1	58	130	5	7	44	9 219	2	9
87	6	21	32	9 604	1	58	131	5	3	25	9 212	2	9
88	6	22	17	9 595	1	58	132	4	59	8	9 204	2	9
89	6	22	55	9 585	1	58	133	4	51	11	9 197	2	9
90	6	23	27	9 576	1	59	134	4	50	3	9 189	2	10
91	6	23	52	9 567	1	59	135	4	45	23	9 182	2	10
92	6	24	9	9 557	1	59	136	4	40	31	9 175	2	10
93	6	24	20	9 548	2	0	137	4	35	49	9 168	2	10
94	6	24	23	9 539	2	0	138	4	30	52	9 161	2	10
95	6	24	20	9 529	2	1	139	4	25	49	9 155	2	10
96	6	24	10	9 520	2	1	140	4	20	51	9 148	2	11
97	6	23	52	9 510	2	1	141	4	15	24	9 142	2	11

142	4	10	5	9 136	2	11	162	2	7	6	9 039	2	14
143	1	1	88	9 129	2	11	163	2	0	17	9 036	2	14
144	3	59	8	9 123	2	11	164	1	53	27	9 033	2	14
145	3	53	81	9 117	2	11	165	1	16	31	9 030	2	14
146	3	47	51	9 112	2	12	166	1	39	35	9 028	2	14
147	3	12	1	9 106	2	12	167	1	32	41	9 025	2	14
148	3	36	13	9 101	2	12	168	1	25	11	9 023	2	11
149	3	30	18	9 095	2	12	169	1	18	39	9 021	2	14
150	3	24	6	9 090	2	12	170	1	11	36	9 020	2	14
151	3	18	13	9 085	2	12	171	1	4	31	9 018	2	14
152	3	12	4	9 080	2	12	172	0	57	26	9 016	2	14
153	3	5	52	9 075	2	13	173	0	50	17	9 015	2	14
154	2	59	31	9 070	2	13	174	0	13	5	9 013	2	14
155	2	53	13	9 066	2	13	175	0	35	55	9 012	2	14
156	2	46	17	9 061	2	13	176	0	28	48	9 011	2	14
157	2	40	19	9 057	2	13	177	1	21	36	9 011	2	14
158	2	33	48	9 054	2	13	178	0	14	24	9 011	2	14
159	2	27	11	9 050	2	13	179	0	7	12	9 010	2	14
160	2	20	33	9 046	2	13	180	0	0	0	9 010	2	14
161	2	13	50	9 043	2	14							

Mean anomaly is  $211^{\circ}-48'-54''$  This is greater than  $180^{\circ}$  and hence the argument will be  $118^{\circ}-11'-6''$  equation for  $148^{\circ}$  is  $3^{\circ}-36'-13''$  and decrease, for  $1^{\circ}$  of argument is  $5'-55''$

$\therefore$  Equation for  $148^{\circ}-11'-6''$  is  $3^{\circ}-35'-7''$  This is negative, as the mean anomaly is greater than  $180^{\circ}$  This when subtracted from the mean longitude gives the true longitude as  $1^{\circ}-6'-47'-58''$  minus  $3^{\circ}-35'-7''$  or  $1^{\circ}-3^{\circ}-12'-51''$  Against the same argument the radius vector and the Hel velocity are read to be as 9 10 and  $2'-12''$  as the Heliocentric velocity

#### POSITION OF NODE

$$\begin{aligned}
 \text{At epoch} &= 3^{\circ}-0^{\circ}-58'-16'' \\
 \text{Annual motion} &= -18' 57'' \\
 \therefore \text{Motion in 112.54 years} &= -112.54 \times -18' 57'' \\
 &= -34'-50'' \\
 \therefore \text{Position of Node at reqd time} &= 3^{\circ}-0^{\circ}-23'-26''
 \end{aligned}$$

#### REDUCTION.

$$\cdot \text{Nodal distance of Saturn (NP)} = 1-3-12-48 \text{ minus}$$

$$3-0-23-26$$

$$= 10-2-49-22$$

$$\text{or } 302^{\circ}-49'-22''$$

$\therefore \tan NM = \cos \epsilon \tan NP$ , where  $\epsilon$  in the case of Saturn's orbit is  $2^{\circ}-49'$

$$\therefore \tan NM = \cos 2^{\circ}-49' \tan 302^{\circ}-49'-22''$$

$$\therefore L \tan NM = L \cos 2^{\circ}-49' + L \tan 302^{\circ}-49'-22'' = 10$$

$$= 9\,999\,4750 + 10\,190\,4276 = 10$$

$$= 10\,189\,9026$$

$$= L \tan 302^{\circ}-51'-16''$$

$$\therefore NM = 302^{\circ}-51'-16''$$

$\therefore$  True Heliocentric longitude corrected for "reduction" is  $302^{\circ}-51'-16''$  plus  $90^{\circ}-23'-26'' = 393^{\circ}-14'-42'' = 1^{\circ}-3^{\circ}-14'-42''$

This could have been also arrived at by the application of the multiplier  $\frac{1}{2}$  to the reduction got from the Moon's table of reduction

### HELIOCENTRIC LATITUDE (P M)

$$\sin PM = \sin \epsilon \sin NP$$

$$= \sin 2^{\circ}-49' \times \sin 302^{\circ}-49'-22''$$

$$\therefore L \sin PM = L \sin 2^{\circ}-49' + L \sin 302^{\circ}-49'-22'' = 10$$

$$= 8\,691\,4379 + 9\,924\,4608 = 10$$

$$= 8\,615\,8987 = L \sin 2^{\circ}-22'$$

$\therefore PM = 2^{\circ}-22'$ . This is negative or south as the Nodal distance is between  $180^{\circ}$  to  $360^{\circ}$

$$SM = SP \cos BM = 9\,1096 \times 999147$$

$$= 9\,1016$$

$$SE = 1\,01646$$

$$\angle MSE = (\angle SM - \angle SE) + 180^{\circ}$$

$$= (33^{\circ}-14'-42'' - 89^{\circ}-21'-22'') + 180^{\circ}$$

$$= 123^{\circ}-53'-20''$$

$$ME^2 = SM^2 + SE^2 - 2 SM SE \cos MSE$$

$$= (9\,1016)^2 + (1\,01646)^2 - 2 \times 9\,1016 \times 1\,01646 \times \cos (123^{\circ}-53'-20'')$$

$$= 82\,8392 + 1\,0332 + 18\,2032 \times 1\,01646 \times \cos (56^{\circ}-6'-40'')$$

$$= 83\,8724 + 18\,5028 \times 5575841$$

$$= 83.8724 + 10.3167$$

$$= 94.1891 \quad \therefore ME = \sqrt{94.1891} = 9.7051$$

$$\begin{aligned} \therefore \sin \hat{SME} &= \frac{SE}{ME} \sin \hat{MSE} \\ &= \frac{1.01616}{9.7051} \times \sin 123^\circ - 53' - 20'' \\ &= \frac{1.01616}{9.7051} \times 8301204 \\ &= .08694234 \\ &= \sin (4^\circ - 59' - 16'') \end{aligned}$$

$$\therefore \angle SME = 4^\circ - 59' - 16''$$

$$\begin{aligned} \therefore \Upsilon' EM &= 33^\circ - 14' - 42'' + \\ &\quad 4^\circ - 59' - 16'' \\ &= 38^\circ - 13' - 58'' = 1^\circ - 8^\circ - 13' - 58'' \end{aligned}$$

## GEOCENTRIC VELOCITY.

$$\text{Geo Vel of Saturn} = \text{Vel of } \angle \Upsilon' SM + \text{Vel of } \hat{SME}$$

$$= \text{Hel velocity} + \frac{SE}{ME} \frac{\cos \hat{MSE}}{\cos \hat{SME}} (-55' - 3'')$$

$$\begin{aligned} \text{for rate of change of } \hat{MSE} &= \text{diff of Hel Vel of Saturn and sun} \\ &= (2' - 11'') - (57' - 14'') = -55' - 3'' \end{aligned}$$

$$\therefore \text{Geocentric Vel of Saturn}$$

$$= (2' - 11'') + \frac{1.01616}{9.7051} \times - \frac{.558}{.996} \times - (55' - .05'')$$

$$= 2' - 11'' + (3' - 12'') = 5' - 54'' \text{ Hence the planet is not retrograde}$$

## GEOCENTRIC LATITUDE (PEM)

$$\tan \text{Geoc Latitude} = \frac{SM}{EM} \tan \text{Hel Latitude}$$

$$= \frac{9.1016}{9.7051} \times \tan 2^\circ - 22'$$

$$= \frac{9.1016}{9.7051} \times 0.413296 = 0.3876$$

$$\therefore \text{Geocentric latitude} = 2^\circ - 19'. \text{ This is also south as the Hel latitude also is of the south direction}$$

# Chapter XVIII.

## URANUS

### ELEMENTS

1	Heliocentric Mean longitude at epoch	5—2—28—48
2	do of Apse	10—29—1—41
3	do of Node	1—21—43—44
4	Length of semi major axis	19 2181
5	Excentricity of orbit	0.0634
6	Inclination of orbit to the ecliptic	46' 4
7.	Periodic time	30686 84 days
8	Annual motion of Apse	= + 3" 22
9	Annual motion of Node	= — 32" 28

In the example no of days from epoch is 41101 82153

Periodic time = 30686 84 days

$$\therefore \text{No of revolutions} = \frac{41101.82153}{30686.84} = 1.339395$$

Neglecting the no of revolutions and converting the decimal to signs etc we get 4°—2'—10"—56 to which if the epoch position 5°—2'—28"—48" be added we get the position at birth to be equal to 9°—4'—39"—44"

Correction due to चरप्राणकलातरदेशातरादि

$$28' - 7'' \text{ is } \frac{28 - 7'' \times 7}{21600} = \text{nil}$$

### FROM THE TABLES

Motion in 40000 days is	3—19—15—24
do 1000 days is	0—11—43—53
do 100 days is	0—1—10—23
do 1 day is	0—0—0—42
do 82153 day	0—0—0—34
Epoch	4—2—10—56
	5—2—28—48

Mean longitude of Uranus at birth = 9—4—39—44



# TABLE OF MEAN MOTION OF URANUS.

Periodic time 30686 84 days

Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds
1	0	0	0	42	300	0	3	31	10	50000	7	16	34	15
2	0	0	1	24	400	0	4	41	33	60000	11	13	53	5
3	0	0	2	7	500	0	5	51	57	70000	3	11	11	56
4	0	0	2	49	600	0	7	2	20	80000	7	8	30	47
5	0	0	3	31	700	0	8	12	43	90000	11	5	49	38
6	0	0	4	13	800	0	9	23	6	100000	3	3	3	29
7	0	0	4	56	900	0	10	33	30	200000	6	6	16	53
8	0	0	5	38	1000	0	11	43	53	300000	9	9	25	27
9	0	0	6	20	2000	0	23	27	46	400000	0	12	33	56
10	0	0	7	2	3000	1	5	11	39	500000	28	15	42	25
20	0	0	14	5	4000	1	16	55	32	600000	6	18	50	53
30	0	0	21	7	5000	1	28	39	26	700000	9	21	59	22
40	0	0	28	9	6000	2	10	23	19	800000	0	25	7	51
50	0	0	35	12	7000	2	22	7	12	900000	3	28	16	20
60	0	0	42	14	8000	3	3	51	5	1000000	7	1	24	49
70	0	0	49	16	9000	3	15	34	58					
80	0	0	56	18	10000	3	27	18	51					
90	0	1	3	21	20000	7	24	37	42					
100	0	1	10	23	30000	11	21	56	33					
200	0	2	20	47	40000	3	19	15	24					

Position of Apse at epoch is

10-29-1-41

Motion in 112 54 years @ 3" 22 per year

6--2

∴ Position of Apse at required time

= 10-29-7-43

Mean anomaly

= { 10-29-7-43 minus  
 9-4-39-44  
 1-24-27-59  
 or 54°-27'-59"

Equation of centre for Uranus in seconds of arc, whose excentricity

$e$  is 04634 is

$19111 \sin nt - 554 \sin 2nt + 22 \sin 3nt$

$\sin 54^\circ - 27' - 59'' = 8137747$

$$\begin{aligned}\sin 2 (54^{\circ}-27-59'') &= \sin 108^{\circ}-55-58'' \\ &= \sin 71^{\circ}-4-2'' = 9458999 \\ \sin 3 (54^{\circ}-27-59'') &= \sin 163^{\circ}-23-57'' \\ &= \sin 16^{\circ}-36-3'' = 2857023\end{aligned}$$

Equation of centre in seconds of arc

$$\begin{aligned}&= 19111 \times 8137747 - 554 \times 9458999 + 22 \times 2857 \\ &= 15552 - 524 + 6 = 15034\end{aligned}$$

In degrees etc the equation is  $4^{\circ}-10-34$

Applying this to the mean longitude we get true longitude

$$\left\{ \begin{array}{r} 9-4-39-41 \text{ plus} \\ 4-10-34 \\ \hline 9-8-50-18 \end{array} \right.$$

The Heliocentric Velocity is found out from the formula  $(42-4 \cos n t)$

In the present case it is  $42-4 \cos 54^{\circ}-28$  in seconds of arc

$$= 42 - 4 \times 58 = 42 - 2 = 40''$$

## RADIUS VECTOR.

$$r = \frac{a(1-e^2)}{1-e \cos \theta} \quad \text{where}$$

$a$  in the case of Uranus is 19 2181

$e$  is 0.4634

and  $\theta$  is as usual the true anomaly

$$\begin{aligned}\text{In this case } \theta &= \left\{ \begin{array}{r} 10-29-7-43 \text{ minus} \\ 9-8-50-18 \\ \hline 1-20-17-25 \\ \text{or } 50^{\circ}-17-25'' \end{array} \right. \\ &\text{or } 50^{\circ}-17-25''\end{aligned}$$

$$\begin{aligned}r &= \frac{19\,2181 \times [1 - (0.4634)^2]}{1 - 0.4634 \cos 50^{\circ}-17-25''} = \frac{19\,1769}{1 - 0.4634 \times \cos 50^{\circ}-17} \\ &= \frac{19\,1769}{1 - 0.4634 \times 639} = \frac{19\,1769}{9704} = 19\,7619\end{aligned}$$

The numerator is always constant it is enough if the denominator alone is worked out and the radius vector arrived at

In the example mean anomaly is  $54^{\circ}-27-59''$

Equation of centre for  $54^{\circ}$  degrees is  $4^{\circ}-9-0''$  and

do for  $55^{\circ}$  degrees is  $4^{\circ}-12-19''$

Equation of centre for  $54^{\circ}-27-29''$  is  $4^{\circ}-10-33''$  which is the same as that already arrived at against the same item the radius vector and Heliocentric velocity are seen to be 19 75 and 40

# TABLE OF EQUATION OF CENTRE, HELIOCENTRIC VELOCITY AND RADIAL VECTOR OF URANUS.

Arg — Mean anomaly If  $> 180^\circ$  the defect from  $360^\circ$  will be the argument but the equation of centre will be negative

Deg	Equat on of Centre	Rad us Vector	Hel Velocity	Deg	Equat on of Centre	Radius Vector	Hel Velocity
0 0 0	0	20 092	38	35	2 54 22	19 942	39
1 0 5	15	20 091	38	36	2 58 47	19 934	39
2 0 10	30	20 091	38	37	3 3 9	19 925	39
3 0 15	45	20 090	38	38	3 7 29	19 915	39
4 0 21	0	20 089	38	39	3 11 43	19 906	39
5 0 26	15	20 088	38	40	3 15 57	19 897	39
6 0 31	29	20 087	38	41	3 20 9	19 888	39
7 0 36	43	20 085	39	42	3 24 14	19 879	39
8 0 41	56	20 082	38	43	3 28 18	19 868	39
9 0 47	8	20 080	38	44	3 32 19	19 858	39
10 0 52	20	20 078	38	45	3 36 15	19 847	39
11 0 57	31	20 076	38	46	3 40 7	19 837	39
12 1 2 40		20 074	38	47	3 43 58	19 826	39
13 1 7 49		20 070	38	48	3 47 13	19 816	39
14 1 12 58		20 067	38	49	3 51 27	19 806	39
15 1 18 5		20 063	38	50	3 55 4	19 796	39
16 1 23 10		20 059	38	51	3 58 40	19 786	39
17 1 28 15		20 056	38	52	4 2 10	19 776	39
18 1 33 18		20 052	38	53	4 5 37	19 766	39
19 1 38 19		20 047	38	54	4 9 0	19 757	40
20 1 43 20		20 041	38	55	4 12 19	19 743	40
21 1 48 17		20 036	38	56	4 15 34	19 728	40
22 1 53 14		20 031	38	57	4 18 44	19 714	40
23 1 58 9		20 026	38	58	4 21 51	19 700	40
24 2 3 2		20 021	38	59	4 24 53	19 686	40
25 2 7 54		20 014	38	60	4 27 50	19 672	40
26 2 12 42		20 007	38	61	4 30 43	19 659	40
27 2 17 29		20 001	38	62	4 33 32	19 645	40
28 2 22 14		19 994	38	63	4 36 16	19 632	40
29 2 26 56		19 988	38	64	4 38 55	19 619	40
30 2 31 38		19 981	39	65	4 41 29	19 605	40
31 2 36 16		19 973	39	66	4 44 0	19 592	40
32 2 40 50		19 965	39	67	4 46 25	19 578	40
33 2 45 24		19 957	39	68	4 48 44	19 563	40
34 2 49 55		19 949	39	69	4 40 59	19 549	40

70	4	53	10	19 535	40	114	4	57	45	18 861	43
71	4	55	16	19 521	40	115	4	55	37	18 846	43
72	4	57	17	19 507	41	116	4	58	27	18 831	43
73	4	59	11	19 492	41	117	4	51	12	18 817	43
74	5	1	1	19 477	41	118	4	48	50	18 803	43
75	5	2	46	19 462	41	119	4	46	23	18 788	43
76	5	4	20	19 447	41	120	4	43	49	18 774	43
77	5	6	0	19 433	41	121	4	41	11	18 760	44
78	5	7	29	19 418	41	122	4	38	26	18 746	44
79	5	8	53	19 403	41	123	4	35	36	18 732	44
80	5	10	10	19 387	41	124	4	32	40	18 718	44
81	5	11	24	19 372	41	125	4	29	40	18 704	44
82	5	12	31	19 457	41	126	4	26	34	18 691	44
83	5	13	34	19 342	41	127	4	23	22	18 675	44
84	5	14	29	19 327	41	128	4	20	4	18 658	44
85	5	15	20	18 311	41	129	4	16	44	18 652	44
86	5	16	5	19 296	41	130	4	13	15	18 639	44
87	5	16	44	19 280	41	131	4	9	41	18 626	44
88	5	17	17	19 265	41	132	4	6	5	18 613	44
89	5	17	46	19 249	41	133	4	2	23	18 601	44
90	5	18	8	19 234	41	134	3	53	34	18 589	44
91	5	18	24	19 218	41	135	3	54	42	18 577	44
92	5	18	35	19 202	41	136	3	50	45	18 565	45
93	5	18	40	19 186	42	137	3	46	43	18 554	45
94	5	18	39	19 170	42	138	3	42	35	18 542	45
95	5	18	32	19 155	42	139	3	38	23	18 531	45
96	5	18	20	19 139	42	140	3	34	8	18 520	45
97	5	18	2	19 123	42	141	3	29	47	18 510	45
98	5	17	36	19 107	42	142	3	25	23	18 499	45
99	5	17	6	19 092	42	143	3	20	54	18 488	45
100	5	16	29	19 076	42	144	3	16	20	18 478	45
101	5	15	48	19 060	42	145	3	11	13	18 470	45
102	5	15	0	19 044	42	146	3	7	2	18 462	45
103	5	14	5	19 029	42	147	3	2	15	18 455	45
104	5	13	6	19 014	42	148	2	57	26	18 444	45
105	5	12	0	18 998	43	149	2	52	33	18 434	45
106	5	10	48	18 983	43	150	2	47	37	18 423	45
107	5	9	31	18 967	43	151	2	42	36	18 415	45
108	5	8	8	18 952	43	152	2	37	32	18 407	45
109	5	6	38	18 937	43	153	2	32	25	18 400	45
110	5	5	2	18 921	43	154	2	27	14	18 392	45
111	5	3	21	18 906	43	155	2	22	2	18 381	45
112	5	1	31	18 891	43	156	2	16	45	18 376	46
113	4	59	41	18 876	43	157	2	11	26	18 370	46

158	2	6	4	18 368	46	170	0	58	39	18.307	46
159	2	0	38	18 357	46	171	0	52	50	18 304	46
160	1	55	12	18 351	46	172	0	47	1	18 301	46
161	1	49	11	18 315	46	173	0	41	10	18 299	47
162	1	44	8	18 339	46	174	0	35	19	18 296	46
163	1	39	34	18 334	46	175	0	29	28	18 295	46
164	1	32	57	18 329	46	176	0	23	35	18 294	46
165	1	27	18	18 325	46	177	0	17	41	18 293	46
166	1	21	38	18 321	46	178	0	11	48	18 292	46
167	1	15	55	18 316	46	179	0	5	54	18 291	46
168	1	10	11	18 312	46	180	0	0	0	18 291	46
169	1	4	26	18 309							

## POSITION OF NODE.

At epoch it is  $1^{\circ}-21'-43''-44''$

Annual motion is  $-33''$  28

$\therefore$  Motion in 112.54 years =  $1^{\circ}-0'-31''$  minus

$\therefore$  Position of Node at birth =  $1^{\circ}-20'-43''-13''$

True Heliocentric longitude of Uranus =  $9^{\circ}-8'-50''-18''$

$\therefore$  Nodal distance (NP) =  $7^{\circ}-18'-7''-5''$

$\tan NM = \cos i \tan NP$ , where  $i = 46'$  for the orbit of Uranus

$\therefore L \tan NM = L \cos i + L \tan NP = 10$

$= L \cos 46' + L \tan 228^{\circ}-7'-5'' = 10$

$= 9.9999611 + 10.0169538 = 10$

$\therefore = 10.0169149$

$\therefore NM = 180^{\circ} + 48^{\circ}-6'-56'' = 228^{\circ}-6'-56''$

$\therefore$  True Hel longitude =  $228^{\circ}-6'-56'' + 50^{\circ}-43'-13'' = 278^{\circ}-50'-9''$

The necessary reduction could have been also arrived at from reduction found out with the help of the Moon's tables and applying the multiplier  $\frac{1}{45}$

## • LATITUDE (P M)

$\sin PM = \sin i \sin NP$

$\sin PM = \sin i \sin NP$

$= \sin 46' \sin 228^{\circ}-7'-5''$

$\therefore L \sin PM = L \sin 46' + L \sin 228^{\circ}-7'-15'' = 10$

$$= 8^{\circ}12'47.10'' + 9^{\circ}8'18.775'' - 10''$$

$$= 7^{\circ}9'8.3485''$$

$\therefore PM = 34'$ , but as NP is more than  $180^{\circ}$ , this Hel latitude is south or negative

$$SM = SP \cos PSM = 19\,7619 \cos 34'$$

$$= 19\,7619 \times 9999511 = 19\,7609$$

$$\angle MSE = \angle \overset{\circ}{\smash{\mathcal{I}}}SM - \angle \overset{\circ}{\smash{\mathcal{I}}}SE = 278^{\circ}-50'-9'' - \overbrace{89^{\circ}-21'-23''}^{+180^{\circ}}$$

$$= 9^{\circ}-28'-47''$$

$$EM^2 = SM^2 + SE^2 - 2 SM \cdot SE \cos \hat{MSE}$$

$$= (19\,761)^2 + (1\,01646)^2 - 2 \times 19\,761 \times 1\,01646 \times \cos (9^{\circ}-28'-47'')$$

$$= 390\,49 + 1\,0332 - 40\,171 \times 98633$$

$$= 391\,5232 - 39\,6218 = 351\,9014$$

$$\therefore EM = \sqrt{351\,9014} = 18\,758$$

$$\sin \hat{SME} = \frac{SE}{EM} \sin \hat{MSE} = \frac{1\,01646}{18\,758} \times \sin 9^{\circ}-28'-47''$$

$$\therefore \hat{SME} = 5^{\circ}-7'-15''$$

$$\therefore \overset{\circ}{\smash{\mathcal{I}}}EM = \overset{\circ}{\smash{\mathcal{I}}}SM + \hat{SME}$$

$$= \overbrace{278^{\circ}-50'-9''}^{+5^{\circ}-7'-15''}$$

$$= 283^{\circ}-57'-24''$$

$$= 9^{\circ}-13^{\circ}-57'-24''$$

This is the Geocentric longitude of Uranus

## GEOCENTRIC VELOCITY.

$$\text{This} = \text{Rate of change of } \overset{\circ}{\smash{\mathcal{I}}}SM + \frac{SE \cos \hat{MSE}}{EM \cos \hat{SME}} \times \text{rate of change of } \angle MSE$$

$$= 40'' + \frac{1\,016}{18\,76} \times \frac{\cos 9^{\circ}-29'}{\cos 5^{\circ}-7'} \times -(58\,44)$$

$$= 40 + \frac{1\,016}{18\,76} \times \frac{.986}{.996} \times -58\,44$$

$$= 40'' - \frac{58\,549}{18\,685} = 40'' - (3'-8'') = -(2'-28'')$$

$\therefore$  The planet is therefore retrograde

## GEOCENTRIC LATITUDE.

$$\begin{aligned}\text{Sine of This is} &= \frac{SP}{EM} \sin \text{Hel lat} = \frac{19\,762}{18\,758} \times \sin 34' \\ &= \frac{19\,762}{18\,758} \times .00989 = .010412\end{aligned}$$

∴ Geocentric latitude = 36'

This is also south or negative according to the direction of the Hel latitude

## Chapter XIX.

### NEPTUNE

#### ELEMENTS

	S	D	M	S
1 Mean Heliocentric longitude at epoch is	6	24	6	20
2 do of Apse	6	24	15	22
3 do of Node	3	18	28	1
4 Length of semi major axis	30	1096		
5 Inclination of orbit to plane of the ecliptic	= 1°	46'	9	
6 Periodic time	60186	64	days	
7 Annual motion of Apse	= 1°	19	(+)	
8 Annual motion of Node	= 10"	68	(—)	
9 Excentricity of the orbit	= .009			

In the example taken no of days from epoch is 41101 82153 days  
Dividing this by the Periodic time we get

$$\text{No of revolutions} = \frac{41101\,82153}{60186\,64} = 682906$$

Converting this to signs etc we get 8°—5°—50—46, to which if the epoch mean longitude 6°—24°—6—20" be added we get mean longitude of Neptune as 2°—29°—57—6' Correction for the observer is nil for

$$\frac{28' - 7 \times 22''}{21600} = \text{nil}$$

# TABLE OF MEAN MOTION OF NEPTUNE.

Periodic time 60186 64 days

Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds	Days	s	Degrees	Minutes	Seconds
1	0	0	0	22	300	0	1	47	40	50000	9	29	4	11
2	0	0	0	43	400	0	2	23	33	60000	11	28	53	1
3	0	0	1	5	500	0	2	59	27	70000	1	28	41	51
4	0	0	1	26	600	0	3	35	20	80000	3	28	30	41
5	0	0	1	45	700	0	4	11	13	90000	5	28	19	31
6	0	0	2	9	800	0	4	47	6	100000	7	28	8	21
7	0	0	2	31	900	0	5	23	0	200000	3	26	16	43
8	0	0	2	52	1000	0	5	58	53	300000	11	24	25	4
9	0	0	3	14	2000	0	11	57	46	400000	7	22	33	26
10	0	0	3	35	3000	0	17	56	39	500000	3	20	41	47
20	0	0	7	11	4000	0	23	55	32	600000	11	18	50	8
30	0	0	10	46	5000	0	29	54	25	700000	7	16	58	30
40	0	0	14	21	6000	1	5	53	18	800000	3	15	6	51
50	0	0	17	57	7000	1	11	52	11	900000	11	13	15	13
60	0	0	21	32	8000	1	17	51	4	1000000	7	11	23	31
70	0	0	25	7	9000	1	23	49	57					
80	0	0	28	43	10000	1	29	48	50					
90	0	0	32	18	20000	3	29	37	40					
100	0	0	35	53	30000	5	29	26	30					
200	0	1	11	47	40000	7	29	15	21					

## FROM TABLES

Motion in 40000 days is 7—29—15—21

do 1000 days is 0—5—58—53

do 100 days is 0—0—35—53

do 1 day is 0—0—0—22

do 82153 of a day is 18

Position at Epoch 6—24—6—20

Correct Mean longitude of Neptune at birth } 2—29—57—7



## POSITION OF APSE.

At epoch	6—24—15—22
Annual motion is $1^{\circ} 19'$ +	
∴ Motion in 11254 years is	0—0—2—14
∴ Position of Apsē at required time	= 6—24—17—36
Mean anomaly	= { 6—24—17—36 minus 2—29—57—6 ————— 3—24—20—30 or 114°—20'—30"

∴ Equation of centre in seconds of arc for Neptune whose eccentricity  $e$  of the orbit is .009, is given by the formula,  
 $3713 \sin nt - 21 \sin 2nt$

In th the present example

$$\begin{aligned}
 \text{Equation of centre} &= 3713 \sin (114^{\circ}-20'-30'') - 21 \sin 2 (114^{\circ}-20'-30'') \\
 &= 3713 \sin 65^{\circ}-39'-30'' - 21 \sin (228^{\circ}-41'-0'') \\
 &= 3713 \times 9111038 + 21 \sin 48^{\circ}-41' \\
 &= 3382.9 + 21 \times .7510721 \\
 &= 3382.9 + 15.77 = 3398.67 = 3399'' = 56'-39''
 \end{aligned}$$

$$\therefore \text{True Hel longitude} = \left\{ \begin{array}{l} 2-29-57-6 \text{ plus} \\ \quad \quad \quad 56-39 \\ \hline 3-0-53-45 \end{array} \right.$$

## RADIUS VECTOR.

This =  $\frac{a(1-e^2)}{1-e \cos \theta}$ , where  $\theta$  = is the true anomaly

$$a = 30.1096$$

$$\text{and } e = .009$$

$$\therefore r = \frac{30.1096 (1-.009) (.991)}{1-.009 \cos \theta} = \frac{30.106}{1-.009 \cos \theta}$$

$$\theta = \left\{ \begin{array}{l} 6-24-17-36 \text{ minus} \\ 3-0-53-45 \\ \hline 3-23-23-51 \end{array} \right.$$

$$= 3-23-23-51 = 113^{\circ}-23'-51''$$

$$\cos \theta = \cos 113^{\circ}-23'-51'' = - \cos 66^{\circ}-36'-9'' = -.3971$$

$$= \frac{30.1066}{1+.009 \times .3971} = \frac{30.1066}{1.003575} = 29.996$$

# TABLE OF EQUATION OF CENTRE OF NEPTUNE AND RADIUS VECTOR.

Arg — Mean anomaly If greater than  $180^\circ$  its defect from  $360^\circ$  will be the argument but the equation of centre will be negative

Deg	Equation of Centre	Radius Vector	Deg	Equation of Centre	Radius Vector	Deg	Equation of Centre	Radius Vector
0 0	0 0	30 317	35 0	35 10		70 0	57 56	
1 0	1 4		36 0	36 2	30 267	71 0	58 19	
2 0	2 8		37 0	36 55		72 0	58 40	30 155
3 0	3 12	30 316	38 0	37 46		73 0	59 0	
4 0	4 16		39 0	38 36	30 260	74 0	59 18	
5 0	5 20		40 0	39 26		75 0	59 36	30 112
6 0	6 23	30 315	41 0	40 16		76 0	59 53	
7 0	7 27		42 0	41 4	30 253	77 1	0 9	
8 0	8 31		43 0	41 52		78 1	0 24	30 129
9 0	9 34	30 313	44 0	42 38		79 1	0 38	
10 0	10 37		45 0	43 25	30 246	80 1	0 50	
11 0	11 40		46 0	44 10		81 1	1 1	30 116
12 0	12 43	30 311	47 0	44 55		82 1	1 12	
13 0	13 45		48 0	45 39	30 239	83 1	1 21	
14 0	14 48		49 0	46 22		84 1	1 29	30 102
15 0	15 51	30 308	50 0	47 4		85 1	1 36	
16 0	16 52		51 0	47 45	30 232	86 1	1 41	
17 0	17 53		52 0	48 26		87 1	1 46	30 089
18 0	18 55	30 304	53 0	49 6		88 1	1 49	
19 0	19 56		54 0	49 44	30 224	89 1	1 52	
20 0	20 57		55 0	50 22		90 1	1 53	30 077
21 0	21 57	30 299	56 0	50 59		91 1	1 53	
22 0	22 56		57 0	51 35	30 214	92 1	1 52	
23 0	23 55		58 0	52 10		93 1	1 50	30 062
24 0	24 55	30 295	59 0	52 45		94 1	1 47	
25 0	25 54		60 0	53 18	30 204	95 1	1 43	
26 0	26 51		61 0	53 50		96 1	1 37	30 048
27 0	27 18	30 288	62 0	54 21		97 1	1 31	
28 0	28 45		63 0	54 52	30 192	98 1	1 23	
29 0	29 42		64 0	55 21		99 1	1 14	30 031
30 0	30 39	30 282	65 0	55 50		100 1	1 4	
31 0	31 31		66 0	56 17	30 180	101 1	0 53	
32 0	32 28		67 0	56 41		102 1	0 11	30 021
33 0	33 23	30 275	68 0	57 9		103 1	0 27	
34 0	34 17		69 0	57 33	30 163	104 1	0 13	

105	0 59 57	30 008	131	0 47 3		157	0 24 26	
106	0 59 40		132	0 46 20	29 901	158	0 23 25	
107	0 59 24		133	0 45 37		159	0 22 25	29 892
108	0 59 5	29 995	134	0 44 52		160	0 21 24	
109	0 58 45		135	0 44 7	29 891	161	0 20 22	
110	0 58 28		136	0 43 20		162	0 19 20	29 827
111	0 58 8	29 982	137	0 42 34		163	0 18 17	
112	0 57 38		138	0 41 45	29 882	164	0 17 15	
113	0 57 14		139	0 40 57		165	0 16 12	29 823
114	0 56 48	29 970	140	0 40 7		166	0 15 8	
115	0 56 22		141	0 39 18	29 871	167	0 14 4	
116	0 55 53		142	0 38 27		168	0 13 0	29 820
117	0 55 26	29 957	143	0 37 35		169	0 11 56	
118	0 54 56		144	0 36 42	29 864	170	0 10 52	
119	0 54 26		145	0 35 49		171	0 9 47	29 817
120	0 53 54	29 945	146	0 34 56		172	0 8 42	
121	0 53 22		147	0 34 2	29 856	173	0 7 37	
122	0 52 48		148	0 33 6		174	0 6 32	29 815
123	0 52 14	29 933	149	0 32 11		175	0 5 27	
124	0 51 38		150	0 31 15	29 849	176	0 4 22	
125	0 51 2		151	0 30 18		177	0 3 17	29 814
126	0 50 24	29 922	152	0 29 20		178	0 2 11	
127	0 49 46		153	0 28 22	29 843	179	0 1 5	
128	0 49 7		154	0 27 24		180	0 0 0	29 814
129	0 48 26	29 911	155	0 26 26				
130	0 47 45		156	0 25 26	29 837			

In the case of Neptune the true Hel Velocity does not vary from the mean Hel velocity viz 22 appreciably and hence this may be taken as the true Hel Vel for all practical purposes

In the example the equation of centre and radius vector can be got from the tables

### POSITION OF NODE.

At epoch it is	$3^{\circ}-18'-28"-1$
Annual motion is	$-10'' 68$
$\therefore$ Motion in 11254 years	$= -20'-2''$
$\therefore$ Position at reqd time	$= 3-18-7-59$
$\therefore$ Nodal distance (NP)	$\left\{ \begin{array}{l} = 3-0-53-45 \text{ minus} \\ 3-18-7-59 \\ \hline = 11-12-45-46 \\ = 342^{\circ}-45'-46 \end{array} \right.$

$$\tan NM = \cos \epsilon \tan NP = \cos 1^\circ - 46' 9'' \times \tan 342^\circ - 45' - 46''$$

$$\therefore L \tan NM = L \cos 1^\circ - 46' 9'' + L \tan 342^\circ - 45' - 46'' = 10 \\ = 9\ 9997896 + 9\ 4917310 = 10 \\ = 9\ 1915206 = L \tan (342^\circ - 46' - 46'')$$

$$\therefore NM = 342^\circ - 46' - 46''$$

$\therefore$  True Hel longitude corrected for 'reduction' is  $342^\circ - 46' - 46''$  plus  $108^\circ - 7' - 59''$  that is  $90^\circ - 54' - 45''$  or  $3^\circ - 0' - 54'' - 45''$

The value of the reduction could be also got from the Moon's reduction table but applying the multiplier 13 to it

## HELIOCENTRIC LATITUDE (P M)

$$\sin PM = \sin \epsilon \sin NP \\ = \sin 1^\circ - 47' \sin 342^\circ - 45' - 46''$$

$$\therefore L \sin PM = L \sin 1^\circ - 47' + L \sin 342^\circ - 45' - 46'' = 10 \\ = 8\ 4980398 + 9\ 4716783 = 10 \\ = 7\ 9647183 = L \sin 31^\circ - 41'$$

$\therefore PM = 31^\circ - 41'$ . This is south as the Nodal distance NP is more than  $180^\circ$

$$SM = SP \cos \angle PSM = 29\ 9996 \cos 31^\circ - 11' \\ = 29\ 9996 \times 9999567 = 29\ 9983$$

$$SE = 1\ 01646$$

$$\angle MSE = \frac{(\hat{r} \hat{SM} - \hat{r} \hat{SE}) \times 180^\circ}{\dots} \\ = (90^\circ - 54' - 45'' - 89^\circ - 21' - 23'') \\ = 181^\circ - 33' - 23''$$

$$EM^2 = SM^2 + SE^2 - 2 SM SE \cos \hat{MSE} \\ = (29\ 9983)^2 + (1\ 01646)^2 - 2 \times 1\ 01646 \times 29\ 9983 \\ \times (\cos 181^\circ - 33' - 23'')$$

$$= 899\ 898 + 1\ 0332 + 60\ 984 \times 0.9963$$

$$= 900\ 9312 + 60\ 961.3$$

$$= 961\ 8925 \quad \therefore ME = \sqrt{961\ 8925} = 31\ 0144$$

$$\text{Now } \sin \angle SME = \frac{SE}{ME} \sin \angle MSE$$

$$= \frac{1\ 01646}{31\ 0144} \sin 181^\circ - 33' - 23''$$

$$= \frac{1\ 01646}{30\ 0144} \times -0.271607$$

$$= -0.008902 \quad \therefore \hat{SME} = - (0^\circ - 3' - 4'')$$

105	0 59 57	30 008	181	0 47 3		157	0 24
106	0 59 40		182	0 46 20	29 901	158	0 28
107	0 59 24		183	0 45 37		159	0 22
108	0 59 5	29 995	184	0 44 52		160	0 21
109	0 58 45		185	0 44 7	29 891	161	0 20
110	0 58 28		186	0 43 20		162	0 19
111	0 58 8	29 982	187	0 42 34		163	0 18
112	0 57 38		188	0 41 45	29 882	164	0 17
113	0 57 14		189	0 40 57		165	0 16
114	0 56 48	29 970	140	0 40 7		166	0 15
115	0 56 22		141	0 39 18	29 871	167	0 14
116	0 55 53		142	0 38 27		168	0 13
117	0 55 26	29 957	143	0 37 35		169	0 11
118	0 54 56		144	0 36 42	29 861	170	0 10
119	0 54 26		145	0 35 49		171	0 9
120	0 53 54	29 945	146	0 34 56		172	0 8
121	0 53 22		147	0 34 2	29 856	173	0 7
122	0 52 48		148	0 33 6		174	0 6
123	0 52 14	29 933	149	0 32 11		175	0 5
124	0 51 38		150	0 31 15	29 849	176	0 4
125	0 51 2		151	0 30 18		177	0 3
126	0 50 24	29 922	152	0 29 20		178	0 2
127	0 49 46		153	0 28 22	29 843	179	0 1
128	0 49 7		154	0 27 24		180	0 0
129	0 48 26	29 911	155	0 26 26			
130	0 47 45		156	0 25 26	29 837		

In the case of Neptune the true Hel Velocity does not v mean Hel velocity viz 22 appreciably and hence this may be true Hel Vel for all practical purposes

In the exampmle the equation of centre and radius vecto from the tables

### POSITION OF NODE.

At epoch is it is	3°—18°—26
Annual mot on is	—10° 66
Motion in 11254 years	= —20 —
Position at reqd time	<sup>s</sup> = 3—18—7
Nodal distance (NP)	{ = 3—0—58 3—18—7 = 11—12— = 342°—4'

To have a correct estimate of the nature of the influences of these planets their correct positions with their mutual positions also and with respect to the observer their visibility or invisibility their brilliancy or combustion have all to be taken fully into account and only the correct time will determine all these

While the importance of the correct instant is so much let us see what the actual conditions afford us in the matter of finding out the correct instant

In olden days the time during day was found out by measuring the length of shadow cast by a gnomon at any required time and by computing a spherical  $\Delta$ , with the length of the shadow the latitude of the place and the sun's declination on the day in question This will give the correct apparent solar time from sunrise or for sunset

During the nights, the star crossing the meridian of the place is noted and its RA being known is diminished by the sidereal time at noon on the day The balance will be the hours mts secs since the proceeding noon in sidereal hours which can be easily converted into mean hours by subtracting at the rate of 3 mts 56 sec for every 24 sidereal hours

During cloudy days or nights when neither the sun nor the star could be visible they used to have (जलयंत्र) a light metallic cup with a thin fine hole, capable of floating in water, but when water enters into and reaches a definite mark the cup will sink with a tick The interval between two consecutive sinkings will be taken as the unit of time which is previously got ascertained with either of the previous two methods To allow for the loss of time between sinking and refloating two such contrivances are had which may be used alternately

These seem to be very crude and unrefined in the modern times of watches and clocks and wristlets These are able to show only correct to a minute, which will result in a difference of 15' of arc whereas all the previous methods will exactly measure fractions of the unit employed The Hindu has for his unit of time a *ughatika* (विघटिका), which is equal to 6' of arc and it will be found to be  $2\frac{1}{2}$  times nearer than the modern unit of time

Even if the exact moment be possible to be noted by some process there is yet a wide contest as to what really is the act of birth whose time is to be noted —

1) Whether the appearance of the child out of the mother's body

$$\begin{aligned}
 \therefore \text{Geocentric longitude} &= \hat{\gamma} \cdot \hat{SM} + \hat{SME} \\
 &= 90^\circ - 54' - 45'' \text{ minus } 0^\circ - 3' - 1'' \\
 &= \underline{90^\circ - 51' - 41''}
 \end{aligned}$$

## GEOCENTRIC VELOCITY.

$$\begin{aligned}
 \text{This} &= \text{Hel velocity} + \frac{SE \cos \angle MSE}{ME \cos \angle SME} \times \text{Rate of change of MSE} \\
 &= 22'' + \frac{1\ 01646}{31\ 0144} \times -\frac{99969}{31} \times -\left(56' - 52''\right) \\
 &= 22' + 1' - 50'' = \underline{2' - 12''}
 \end{aligned}$$

## GEOCENTRIC LATITUDE

$$\begin{aligned}
 \tan (\text{Geo Latitude}) &= \frac{SM}{EM} \sin (\text{Hel Latitude}) \\
 &= \frac{29\ 9996}{31\ 0144} \times \sin 32' = \frac{29\ 9996}{30\ 0144} \times 0093 \\
 &= 009 = \tan 31' \\
 \therefore \text{Geocentric latitude} &= 31'. \text{ S as the Hel latitude is also south}
 \end{aligned}$$

# Chapter XX.

## RECTIFICATION OF BIRTH TIME.

It is needless for me to reiterate the importance of the exact moment of birth when a being is ushered into this planet to work out its destiny. As a body is born it comes to be under the action of the forces of the earth and has to undergo all the changes consequent to the forces real and virtual to which the planet itself is subject to due to other planets and the Sun.

Besides the planets themselves are not always exerting the same nature and amount of forces for their position relative to the sun and the earth on one side and relative to other planets on the other as also the portion of the earth's surface presented to the planets at the instant receiving different kinds of reflected radiant energy from the planets will be different at different times.

To have a correct estimate of the nature of the influences of these planets their correct positions with their mutual positions also and with respect to the observer their visibility or invisibility their brilliancy or combustion have all to be taken fully into account and only the correct time will determine all these

While the importance of the correct instant is so much let us see what the actual conditions afford us in the matter of finding out the correct instant

In olden days the time during day was found out by measuring the length of shadow cast by a gnomon at any required time and by computing a spherical  $\Delta$  with the length of the shadow the latitude of the place and the sun's declination on the day in question This will give the correct apparent solar time from sunrise or for sunset

During the nights the star crossing the meridian of the place is noted and its RA being known is diminished by the sidereal time at noon on the day The balance will be the hours mts secs since the proceeding noon in sidereal hours which can be easily converted into mean hours by subtracting at the rate of 3 mts 56 sec for every 24 sidereal hours

During cloudy days or nights when neither the sun nor the star could be visible they used to have (जलयन्त्र) a light metallic cup with a thin fine hole capable of floating in water but when water enters into and reaches a definite mark the cup will sink with a tick The interval between two consecutive sinkings will be taken as the unit of time which is previously got ascertained with either of the previous two methods To allow for the loss of time between sinking and refloating two such contrivances are had which may be used alternately

These seem to be very crude and unrefined in the modern times of watches and clocks and wristlets These are able to show only correct to a minute which will result in a difference of 15 of arc whereas all the previous methods will exactly measure fractions of the unit employed The Hindu has for his unit of time a *vighatika* (विघटिका), which is equal to 6 of arc and it will be found to be 2½ times nearer than the modern unit of time

Even if the exact moment be possible to be noted by some process there is yet a wide contest as to what really is the act of birth whose time is to be noted —

1) Whether the appearance of the child out of the mother's body



2) Whether the contact on this planet

3) Whether the cutting off of the umbilical cord when the child commences the breath

Or

4) Whether the time of first cry which always accompanies the first breath except in cases of still born ones

Each is equally important as the other and at any moment all cannot be correct. There should be some other thing with reference to which alone the time of the moment of the birth could be found out

The exact moment nearer to an unit less than a minute has no importance as the planetary positions except that of the Moon in whose case it may result about 33' of arc will not be much affected. But for viewing the effects of planets especially in Hindu Astrology each planet is supposed to be changing its occupation or inclination to good or bad for every  $\frac{1}{16}$  of a vighatika or  $\frac{1}{4}$  of a minute.

A fitting example can be given here at this stage. Two persons were born to two different mothers at an interval of 2 minutes or so. Due to the short interval the ascendant and other houses planetary positions mutual aspects on the houses and the like could not have changed so much as to admit of any diversity in the growth environments culture prospects and attainments of the two individuals. In due course one became a District Judicial officer while the other was a revered Village Brahmin preceptor. How could this difference be explained from the practically similar birth charts differing only by a time interval of 2 minutes?

This was a great perplexity for a long time until it was explained that the change has been wrought up by the change in the planetary occupation due to the small increase of time. The lord of avocation in each of the two cases was considered and also the planetary occupation. For the former it was Ascending the throne and that for the latter Making sacred offering. This gives the clue to the inclination of the two natives.

Experience and research have proved beyond any doubt that the conception is brought about by the influence of the Moon on the male and female potency. The Moon our neighbour and satellite is always considered as the source for all impregnation and it is only the Moon which will give us the clue to the determination of the correct time of birth.

Therefore the time of conception is usually worked out from the given time of birth supposed to be incorrect by a laid out method which was

hypothetical at first, but later on found to be true and hence established and thence the time of birth is rectified by going backward or forward to make the Moon and ascendant at birth and conception adjust themselves according to certain formulated principles

The method to be mentioned herein has no reference to the time of conception yet it gives the rectification. It may appear to be hypothetical also but practical working will show that it will give very satisfactory results in a much simpler method than the "Prenatal Epoch Theory"

It was a mis-conception among the Hindus that the European astronomers were the foremost to formulate the Prenatal Theory of Rectification of birth time and that our ancient Indian astronomers have not striven to that end with any success.

The source I am quoting herein is a half-sloka from *Varahamira's Brihatjataka* one of the ancient authorities on Astrology. The sloka has been laid out to be used for predicting the probable time of birth from the time conception or query. Different commentators have given different meanings but not one has made even a passing mention of its application for the purpose of rectification of birth time.

A very deep research was undertaken and a method has been evolved out of it which is explained in the following paragraphs

The sloka reads as follows —

तत्कालमिदुसहितो द्विरसाशको यः ।

तत्तुल्य राशिसहिते पुनः शशके ॥

....

....

....

(vide sloka Chapter IV of *Brihatjataka*)

The meaning of this is — Find out the particular dwadasamsa of the Moon in any particular sign which it may be occupying. Note it count from the next sign as many signs as the no. of Dwadasamsas the moon might have passed in the sign taken at first. When the moon comes to that last sign the birth will occur.

(Note — Dwadasamsa — means the twelfth part of a sign)

The principle contained in applying this sloka is that the birth lagna or ascendant is worked upon according to the instructions in sloka. For, the sloka requires the position of moon at conception which will be the ascendant at birth or its opposite. The Hindus knew the interchangability of the

Moon at conception or its seventh with that of ascendant at birth or its seventh.

On working out according to the instructions given, a particular position of Moon will be arrived at. Compare the position of the Moon thus arrived-at with that of the Moon for the noted time of birth, which we shall hereafter call the *observed Moon*.

By a comparison of the two moons, we deduce the correction to be applied to the ascendant to make the two moons tally exactly.

In the example of this book, the position of Ascendant is  $0^{\circ}-28^{\circ}-44'-56''$  and that of the Moon is  $3^{\circ}-8^{\circ}-42'-8''$

To find out the number of Dwadasamsas passed by the ascendant, convert the degrees etc to seconds of arc and dividing it by 9000 the number of seconds of arc in a dwadasamsa, we get  $\frac{103496}{9000} = 11\frac{4496}{9000}$  dwadasamsas. Now as per the instructions, we have to find the sign where this dwadasamsa falls, and from the sign next to that, to count again as many sign as the dwadasamsas. Therefore, converting the ascendant also to signs, we get  $\frac{103496}{108000}$  signs (If the ascendant has in its longitude any signs, the no. of signs should be prefixed here) Hence the sign where the Moon at birth should be, will fall at

$$\frac{103496}{108000} + 2 \left( 11 \frac{4496}{9000} \right) + 1 = 23 \frac{4496}{4500} + \frac{103496}{108000}$$

$$= 23 + \frac{107904 + 103496}{108000} = 23 + \frac{2114}{1080} = 24 \frac{1034}{1080}$$

subtracting 12, as many times as possible being revolutions, we get it as  $\frac{1034}{1080}$  signs or  $0^{\circ}-28^{\circ}-43'-20''$ . This should be the position of Moon at the observed moment, whereas it is  $3^{\circ}-8^{\circ}-42'-8''$ . Therefore there is correction to be applied, as the arrived-at Moon is within  $90^{\circ}$  of the observed Moon the position is called an operating one and if it is nearer to the seventh from Moon, it will be a separating one,

In the case of operating positions, the twice the no of dwadasamsas are counted from the sign next to ascendant and in the case of separating positions, the twice the no of dwadasamsas are subtracted from the sign next to ascendant and the resultant thus got is taken for comparison with the observed moon, to get the time correction.

In the present instance, the case is one of operating influence and therefore the counting of the dwadasamsas in the normal order of the signs done by us is correct. Therefore the arrived-at moon lags behind the

observed moon and hence observed time is behind the correct time or the correction to be applied to the ascendant will therefore be additive.

Let  $x^\circ$  be the change in the lagna, to be added to the original ascendant. The increase in the L. H. S of the equation already given will be  $\left(\frac{x}{30} + \frac{x \times 2 \times 2}{5}\right)$  signs. In the interval of  $x^\circ$  of ascendant, the Moon also would have increased by  $\left(\frac{x \times 4 \times \text{Moon's daily Velocity in degrees}}{60 \times 24 \times 30}\right)$  signs [for,  $x^\circ$  arc correspond to  $4x$  mts of time. Strictly the time should be found out by dividing the product of the duration of the tropical sign and  $x$  by 30]

Therefore we have,

$$1080 \frac{1034}{1000} + \frac{5x}{6} = 3 \frac{13328}{108000} + \frac{x \times 4 \times 911.75}{60 \times 24 \times 60 \times 30}$$

$$\therefore \frac{5x}{6} - \frac{911.75 \times x}{648000} = 3 \frac{13328}{108000} - \frac{1034}{1000} \quad 1080$$

Multiplying throughout by the L C M 648000, we get,

$$(540000 - 911.75)x = 1944000 + 79938 - 372240$$

$$= 1651728$$

$$\therefore 539088.25 \times x = 1651728$$

$$\therefore x = \frac{1651728}{539088.25} = \frac{2^\circ - 36' - 13''}{2-36-13} \quad (\text{use this for further})$$

$\therefore$  Applying the correction which has already been stated to be additive, the observed ascendant, we get

$$\therefore \text{True lagna} = \begin{matrix} 0^\circ - 28' - 44'' - 56'' \text{ plus} \\ \underline{2^\circ - 36' - 13''} \\ \hline \end{matrix}$$

$$\text{Rectified ascendant} = \begin{matrix} 1^\circ - 1' - 46'' - 46'' \\ \underline{2^\circ - 36' - 13''} \\ \hline \end{matrix}$$

## EXAMPLE FOR A SEPARATING INFLUENCE.

Let the Observed Ascendant  $0^\circ - 19' - 51'' - 43''$  and the observed Moon  $9^\circ - 14' - 10'' - 26''$

As before the Dwadasamsa of the lagna is  $\frac{71503}{90000} = 78\frac{503}{90000}$ . Twice this is  $15\frac{4003}{45000}$  Adding one sign and counting from the lagna or ascendant, we

$$\text{get that the Moon should be at } 15\frac{4003}{45000} + 1 + \frac{71503}{108000}$$

$$= 16 + \frac{96072 + 71503}{108000} = 17\frac{59575}{108000}, \text{ subtracting from this an exact no.}$$

those for the next three tropical signs will be

Cancer	Leo	Virgo
322	298.8	279 2
+ 87	+ 21 3	+ 26 3
330 7	320 1	305 5

The same figures read backwards will give the durations of the remaining six tropical houses.

Now write out the tropical longitudes of all the twelve houses for sake of ready reference, from pages 87 and 88

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
s	1	2	3	4	5	6	7	8	9	10	11	0
°	21	19	15	12	12	17	21	19	15	12	12	17
'	21	18	11	13	49	2	21	18	11	13	49	2
"	44	14	26	7	<del>47</del>	18	44	14	26	7	<del>11</del>	18

Now the rectified ascendant is  $1^{\circ}-1^{\circ}-48'-46''$  Making it tropical, we get, by adding precession  $22^{\circ}-36'-48''$  the tropical ascendant to be  $1^{\circ}-24^{\circ}-25'-34''$ . The difference is  $3^{\circ}-3'-50''$ . The tropical ascendant is in Taurus, whose duration is for 277.5 vighatkas. If this is for  $30^{\circ}$  of arc, what will correspond to  $3^{\circ}-3'-50''$ ? This is simple rule of three. It will be  $\frac{3^{\circ}-3'-50'' \times 277.5}{30^{\circ}}$  vighatkas

$$\text{or } \frac{11030 \times 277.5}{108000} = \frac{122433}{4320} = 28\ 341 \text{ vighatkas}$$

∴ This is the time difference by which the ascendant has progressed

## CORRECTION TO SECOND HOUSE.

This tropical house falls in Gemini, whose duration is 313.3 vighatkas. Correction in arc for a time increase of 28.341 vighatkas will be,

$$\frac{28\ 341}{313.3} \times 30^{\circ} = 2^{\circ}-12'-50''$$

of revolutions we get  $5 \frac{7^0 7' 55''}{108000}$ . This should be the moon at the observed instant. Whereas she is found to be nearer the opposite of the arrived at Moon. Hence this is a case of separating influence. Therefore the count will have to be made as a subtractive one from the ascendant increased by one sign.

Therefore the observed moon should be at or near to  $1 \frac{71^0 03' - 154^0 00'}{45000}$ . As the first term is smaller than the latter the first may be increased by multiples of 12 signs to admit of the subtraction. We get  $25 \frac{71^0 03' - 154^0 00'}{108000} = 9 \frac{34^0 31'}{108000}$ . This is the arrived-at Moon. This when converted to signs etc gives  $9^s - 23' - 10'' - 31'''$ . The observed moon is  $9 \frac{51^0 26'}{108000}$ . The difference between the two is  $1 \frac{19^0 41'}{108000}$  of a sign. As before the observed moon is greater than the arrived-at Moon. Hence the correction should be additive had it been an *cp'rating* influence. It being a separating one though the observed moon is more than the arrived-at Moon the correction will be negative and *vice versa*.

If  $x^s$  is the difference to the correction  $\frac{5x}{6}$  signs will be the difference in the results

$$\therefore \frac{5x}{6} = \frac{92405}{108000} \quad \therefore x = \frac{6481}{18000} = 5' 50''$$

Subtractive correction =  $0^s - 21' - 36''$

$\therefore$  The correct lagna will therefore be  $9^s - 19' - 20' - 7''$ . Had the moon's change of longitude been taken into account also the correction would have been still more accurate. It was not done this instance for want of the velocity of the Moon at the instant.

Now we shall find out the time difference and apply it to correct the planetary and house positions to the rectified birth time.

The durations of the signs (tropical) Aries Taurus etc will be in the order 279 2 298 8-322 298 8 279 2 279 2 298 8 322 322 298 8 and 279 2 measured in *Vighatikas* at the equator. It will be shown how these figures are arrived at the end of the chapter.

At any desired latitude we find the durations of these tropical houses as follows. Take the *चरख*'s of the latitude they are 26 3 21 3 and 8 7 for the latitude of the present example.

The durations of the first three tropical signs will be

Aries	Taurus	Gemini
279 2	298 8	322
- 26 3	- 21 3	- 8 7
252 9	277 5	313 3

those for the next three tropical signs will be

Cancer	Leo	Virgo
822	298 8	270 2
+ 87	+ 21 3	+ 26 3
890 7	320 1	305 5

The same figures read backwards will give the durations of the remaining six tropical houses

Now write out the tropical longitudes of all the twelve houses for sake of ready reference from pages 87 and 88

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
s	1	2	3	4	5	6	7	8	9	10	11	0
°	21	19	15	12	12	17	21	19	15	12	12	17
'	21	18	11	13	49	2	21	18	11	13	49	2
."	44	14	26	7	11 7	18	44	14	26	7	11 7	18

Now the rectified ascendant is  $1^{\circ}-1^{\circ}-48-46$  Making it tropical we get by adding precession  $22-36-48$  the tropical ascendant to be  $1^{\circ}-24^{\circ}-25-34$  The difference is  $3^{\circ}-3-50$  The tropical ascendant is in Taurus whose duration is for 277 5 vighatkas If this is for  $30^{\circ}$  of arc what will correspond to  $3^{\circ}-3'-50''$  ? This is simple rule of three It will be  $\frac{3^{\circ}-3-50'' \times 277.5}{30^{\circ}}$  vighatkas

$$\text{or } \frac{11050 \times 277.5}{108000} = \frac{122493}{4820} = 28.341 \text{ vighatkas}$$

This is the time difference by which the ascendant has progressed

## CORRECTION TO SECOND HOUSE.

This tropical house falls in Gemini whose duration is 313 3 vighatkas Correction in arc for a time increase of 28.341 vighatkas will be  $\frac{28.341}{213.3} \times 30^{\circ} = 2^{\circ}-42-50''$

## CORRECTION TO THIRD HOUSE.

This tropical house falls in Cancer, whose duration is 330 7 vighatikas  
 Correction in arc for the same time increase of 28 341 vighatikas will be,  

$$\frac{28\ 341 \times 36^\circ}{330\ 7} = 2^\circ - 34' - 16''$$

## CORRECTION TO FOURTH HOUSE.

This tropical house falls in Leo, whose duration is 320 1 vighatikas  
 $\therefore$  Correction in arc 
$$\frac{28\ 341 \times 36^\circ}{320 \cdot 1} = 2^\circ - 39' - 23''$$

## CORRECTION TO FIFTH HOUSE

This tropical house falls in Virgo whose duration is 305 5 vighatikas  
 $\therefore$  Correction in arc 
$$= \frac{28\ 341 \times 36^\circ}{305\ 5} = 2^\circ - 16' - 59''$$

As the durations of the tropical signs from Aries to Virgo are the same for Libra to Pisces read backwards, it is enough if we take any six houses falling in any of the two sets of six signs. This will save us from the necessity of applying the correction to all the twelve houses. In this instance we have taken all the houses falling in the signs Taurus to Virgo and if we also take the twelfth house, whose tropical longitude falls in Aries we can write down the rectified longitudes of the other houses by merely adding 6 signs to the first set of houses

## CORRECTION FOR TWELFTH HOUSE.

This tropical house falls in Aries whose duration is 252 9 Vighatikas,  
 $\therefore$  Correction in arc 
$$= \frac{28\ 341 \times 36^\circ}{252\ 9} = 3^\circ - 21' - 48''$$

Now applying these corrections to the respective house longitudes (Nirayana) we get them as follows tabulated as under.

### RECTIFIED HOUSE LONGITUDES (NIRAYANA) भावसुदृढः

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
$^\circ$	1	1	2	8	4	5	7	7	8	9	10	11
$'$	1	29	25	22	22	27	1	29	25	22	22	27
$''$	48	24	8	15	59	47	48	24	8	15	59	47
$'''$	46	16	54	42	22	18	46	16	54	42	22	13
				44	55					41	55	



## CORRECTION TO POSITIONS OF PLANETS.

Write down the planetary longitudes with respective velocities under them. They are

Sun	Moon	Node of Moon	Mars	Mercury	Jupiter	Venus	Saturn	Uranus	Neptune
2	3	11	4	3	7	3	1	9	3
29	3	24	5	23	13	1	8	13	0
21	42	6	56	47	23	54	13	57	51
22	8	56	11	26	16	42	58	24	41
—	—	—	—	—	—	—	—	—	—
57	911	3	34	76 <sup>1</sup> <sub>1</sub>	4	73	5	2	2
13	46	11	11	36 <sup>10</sup>	32	57	54	23	16
(Retrograde)			(Retrograde)			(Retrograde)			

Hence correction for a time difference of 28 341 vighatikas forward will be obtained by multiplying this by the velocity in mts of the planet and dividing the result by 3600 when the result will be the correction in mts of arc. For retrograde planets and the Moon's Node the correction will be negative even though the time correction be + ve

The correction for each planet is as follows —

Sun	Moon	Node	Mars	Mercury	Jupt	Venus	Saturn	Uranus	Neptune
0	7'	0	0	0	0	0	0	0	0
27"	10"	2"	16"	36"	2"	35"	3"	1"	1"
+	+	—	+	+	—	+	+	—	+

RECTIFIED PLANETARY POSITIONS WILL BE.

Sun	Moon	Moon's Node	Mars	Mercury	Jupiter	Venus	Saturn	Uranus	Neptune
2	3	11	4	3	7	3	1	9	3
29	3	24	5	23	13	1	8	13	0
21	49	6	56	49	23	55	14	57	51
49	13	54	27	2	14	17	1	23	52

We shall now show how the durations of the natural tropical houses, Tropical Aries etc are found out

$$= 53.277 = \cos 57^\circ - 15'$$

$\therefore \hat{\gamma}P\beta = 57^\circ - 48'$ . If  $\angle \alpha P\beta = h_2$  we have

$$h_2 = \gamma P\beta - \gamma P\alpha = (57^\circ - 48') - (27^\circ - 55') = 29^\circ - 53'$$

Sunning up we have,

$$h_1 = 27^\circ - 55' \text{ or in time } 4\text{ghats } 39\ 2\text{vighats}$$

$$h_2 = 29^\circ - 53' \text{ or } , \quad 4\text{ghat } 58\ 8\text{vighat}$$

$$h_3 = 90^\circ - (h_1 + h_2)$$

$$= 33^\circ - 12' \text{ or } , \quad 5\text{ghats } 32\ 0\text{vighats}$$

As  $\alpha$   $\beta$  and S are  $30^\circ$   $60^\circ$  and  $90^\circ$  from the point  $\gamma$  the first point of Aries these are the durations of the tropical Aries Taurus etc

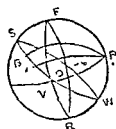
Before finishing this chapter, a passing note has to be made about the ' Prenatal Conception Epoch theory ', wherein the following suggestion may be adopted. A full detailed working and the theorems involved therein are not supposed to be dwelt upon here and they may be had from any standard book on the subject

Only the following change is suggested that after finding out the distance of the moon at epoch above or below the horizon as per the rules in degrees the distance is divided by  $12^\circ$  to find out the day of conception when the Moon will be found in the ascendant or desendant at birth. This 12 is only rough as the Moon's motion is not uniform. The actual day of conception should be found out by first subtracting the period of ten lunar or sidereal periods viz 273days 5hrs 29mts 61sec from the date and time of birth and thence going backward or forward according as the intra-uterine period is more or less than 10 lunar sidereal periods by so many degrees etc of the distance of the Moon above or below the horizon when the actual moment of conception is got. In the case of irregular epochs half a lunar sidereal period will have to be added or subtracted to suit the condition of the case as per the laws of the method

If this is also taken notice of the results fairly tally with the Hindu method of rectification of ascendant



Let  $O$ , be an observer at a place on the terrestrial equator. His celestial equator will be along his prime vertical, as the altitude of the pole, which is the same as the latitude of the place is zero in this case



Let  $SW$  be the plane of the ecliptic, cutting that of the equator at  $\Upsilon$ . Let  $\alpha, \beta$  be the points exactly  $30^\circ$  from  $\Upsilon$  the first point of Aries. Then  $\Upsilon P \hat{L}$  and  $\alpha P \hat{\beta}$  will be the hour angles of the two houses of the zodiac or their duration in sidereal hours. As  $\angle PS\Upsilon = 90^\circ$ ,  $\beta P \hat{S}$  will be the duration of the third house

In  $\Delta \Upsilon P L$  we have

$$\cos \alpha \hat{\Upsilon} = \cos \Upsilon P \cos \alpha P + \sin \Upsilon P \sin \alpha P \cos h_1$$

where  $h_1$  is the duration of the house  $\Upsilon P \alpha (= 30^\circ)$

$$\therefore \cos 30^\circ = \cos 90^\circ \cos \alpha P + \sin 90^\circ \sin \alpha P \cos h_1$$

$$\text{but } \cos 90^\circ = 0, \sin 90^\circ = 1 \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{\sqrt{3}}{2} = \sin \alpha P \cos h_1 \therefore \cos h_1 = \frac{\frac{\sqrt{3}}{2}}{\sin \alpha}$$

$$\text{but } \alpha P = (\text{declination of } \alpha + 90^\circ)$$

$$= (d_1 + 90^\circ)$$

$$\therefore \sin \alpha P = \sin (90^\circ + d_1) = \cos d_1$$

$$\therefore \cos h_1 = \frac{\frac{\sqrt{3}}{2}}{\cos d_1} = \frac{\frac{\sqrt{3}}{2}}{2\sqrt{1-\sin^2 d_1}}$$

but  $\sin d = \sin \omega \sin \odot$  giving the declination ( $d$ ) in terms of the ropical longitude  $\odot$

$$\begin{aligned} \therefore \cos h_1 &= \frac{\frac{\sqrt{3}}{2}}{2\sqrt{1-\sin^2 \omega \sin^2 \odot}} = \frac{\frac{\sqrt{3}}{2}}{2\sqrt{1-\sin^2 \omega \times \frac{1}{4}}} \\ &= \frac{\frac{\sqrt{3}}{2}}{\sqrt{4-\sin^2 \omega}} = \frac{\frac{\sqrt{3}}{2}}{\sqrt{4-(.3980821)^2}} = \frac{1.732}{1.96} \\ &= .8837 = \cos 27^\circ - 55' \end{aligned}$$

$$\therefore h_1 = 27^\circ - 55'$$

similarly,  $\cos \Upsilon P \hat{\beta} = 0 + \sin \beta P \cos \Upsilon P \hat{\beta}$

$$\cos 60^\circ = \frac{1}{2} = \cos d_2 \cos \Upsilon P \hat{\beta}$$

$$\therefore \cos \Upsilon P \hat{\beta} = \frac{1}{2 \cos d_2} = \frac{1}{2\sqrt{1-\sin^2 \omega \sin^2 60^\circ}}$$

$$= \frac{1}{2} \frac{1}{\sqrt{1-\frac{3}{4}\sin^2 \omega}} = \frac{1}{\sqrt{4-3\sin^2 \omega}} = \frac{1}{\sqrt{4-3 \times (.3980821)^2}}$$

$$= 58277 = \cos 57^\circ - 46$$

$$\cdot \hat{\gamma} P\beta = 57^\circ - 48' \quad \text{If } \angle aP\beta = h_2 \text{ we have}$$

$$h_2 = \gamma P\beta - \gamma Pa = (57^\circ - 48') - (27^\circ - 55') = 29^\circ - 53'$$

Sunning up we have

$$h_1 = 37^\circ - 55' \text{ or in time } 4\text{ghs } 39\text{ 2vghats}$$

$$h_2 = 29^\circ - 53' \text{ or } 4\text{ghat } 58\text{ 8vghat}$$

$$h_3 = 90^\circ - (h_1 + h_2)$$

$$= 33^\circ - 12' \text{ or } 5\text{ghats } 32\text{ 0vghalkas}$$

As  $\alpha$   $\beta$  and  $S$  are 30 60 and 90 from the point  $\gamma$  the first point of Aries these are the durations of the tropical Aries Taurus etc

Before finishing this chapter a passing note has to be made about the Prenatal Conception Epoch theory wherein the following suggestion may be adopted A full detailed working and the theorems involved therein are not supposed to be dwelt upon here and they may be had from any standard book on the subject

Only the following change is suggested that after finding out the distance of the moon at epoch above or below the horizon as per the rules in degrees the distance is divided by 12° to find out the day of conception when the Moon will be found in the ascendant or descendant at birth This 12 is only rough as the Moon's motion is not uniform The actual day of conception should be found out by first subtracting the period of ten lunar or sidereal periods viz 273days 5hrs 29mts 61sec from the date and time of birth and thence going backward or forward according as the intra uterine period is more or less than 10 lunar sidereal periods by so many degrees etc of the distance of the Moon above or below the horizon when the actual moment of conception is got In the case of irregular epochs half a lunar sidereal period will have to be added or subtracted to suit the condition of the case as per the laws of the method

If this is also taken notice of the results fairly tally with the Hindu method of rectification of ascendant



# Chapter XXI.

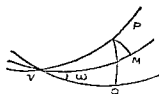
## TRANSFORMATION OF CO-ORDINATES.

The longitude and latitude which we have found out in the previous chapters are with reference to the ecliptic. As stated previously in the earlier chapters the same celestial body can also be located with reference to the celestial equator provided the respective co-ordinates are given. Such a transformation is called The Transformation of co-ordinates — a very important portion in Spherical Astronomy.

### EXAMPE

Let  $P$  be a body whose longitude  $\gamma M$  and latitude  $PM$  are known and let it be required to know the corresponding Right Ascension  $\gamma Q$  and Declination  $PQ$ .

Thro  $P$  draw the vertical, meeting the equator at  $Q$  then  $\gamma Q$  is the RA and  $PQ$  the declination. Join  $P \gamma$ .



In the spherical rt  $\angle d \triangle PM$  we have  $\cos$

$$\gamma P = \cos PM \cos \gamma M$$

$\therefore \gamma P$  is known

$$\text{Further } \sin \gamma M = \tan PM \tan P \gamma M$$

$\therefore P \gamma M$  is known

and  $\angle M \gamma Q$  is already known as  $\omega$

$\therefore P \gamma Q = \angle M \gamma Q + \angle P \gamma M$  which can be found out

In  $\triangle P \gamma Q$   $\gamma P$  and  $\angle P \gamma Q$  are known therefore  $\gamma Q$  and  $PQ$  can be found as follows —

$$\sin PQ = \sin \angle P \gamma Q \sin \gamma P$$

$$= \sin \gamma P \sin (\omega + P \gamma M) \text{ ————— (1)}$$

$$\text{and } \cos \angle P \gamma Q = \cot \gamma P \tan \gamma Q$$

$$\therefore \tan \gamma P = \cos \angle P \gamma Q \tan \gamma Q \text{ ————— (2)}$$

Thus  $\gamma Q$  and  $PQ$  are both found out as the RA and declination of the body  $P$ .

These are very useful to find out the times of rising setting etc of the planets. We propose to make use of these to find out if a planet is combust

with the sun or not To do this we have to find out the hour-angle at rising of the planet and that of the sun and if the difference in their times of rising or setting be found to be less than the values to be mentioned hereafter, which are the values within which the rays of the planets are hidden in those of the sun we call the planet to be in combustion or मोक्ष्य or अस्तंगत in sanskrit

The RA can be found out for all the planets and the difference between each of them and that of the sun will give the time of rising of the planet after the rising of the sun and so also for the setting after that of sun This difference if it falls short of the constant value which will be mentioned now we can conclude that the planet is in combust If more the planet is illumined and bright

For this purpose it is enough if only those planets which are with or in close nearness to the sun are considered and not all the planets In the present example the longitudes of Venus and Sun are seen to be very near differing only about  $2\frac{1}{2}$  nearly Hence here is a case for trying if she is in combust

### EXAMPLE

Longitude of Venus  $91^{\circ}-55'-17''$ , Geoc Latitude  $57'-14''$  N

Longitude of Sun  $89^{\circ}-21'-49''$

Making these tropical we have to add  $22^{\circ}-30'-48''$  the precession We get therefore

Tropical longitude of Venus  $114^{\circ}-32'-5''$

do of Sun  $111^{\circ}-58'-37''$

$$\cos P = \cos 114^{\circ}-32'-5'' \cos 0^{\circ}-57'-14''$$

$$\therefore L \cos P = L \cos 114^{\circ}-32'-5'' + L \cos 57'-14'' - 10$$

$$= 9.6187040 + 9.9999398 - 10$$

$$= 9.6182438 = L \cos 114^{\circ}-31'-52''$$

$$\therefore P = 114^{\circ}-31'-52''$$

$$\cot P \hat{M} = \sin 114^{\circ}-32'-5'' \cot 57'-14''$$

$$L \cot P \hat{M} = L \sin 114^{\circ}-32'-5'' + L \cot 57'-14'' - 10$$

$$= 9.9589029 + 11.7785965 - 10$$

$$= 11.7374994 = L \cot 1^{\circ}-2'-55''$$

$$\therefore PM = 1^{\circ}-2'-55'' (+) \quad \text{This is North as the original lat is North}$$

$$\therefore P / Q = (23^{\circ}-27'-30'' + 1^{\circ}-2'-55'')$$

$$= 24^{\circ}-30'-25''$$

The Ptolemaic Theory stated that the Earth was at the centre of the universe and that the sun and other planets are performing their ceaseless journey about the Earth. This had the consent of the ancient mythologies of the world and the force of religious authorities with their usual oppression was so much that they could not even entertain any theory which was otherwise.

This theory could not explain the theory of Retrograde motion and stationary positions of planets and such other things as easily or rationally as the present Copernican theory does.

Thanks to the Copernican theory the Kepler's Laws and later on Newton's discoveries a great headway has been made in the field and things have been established on a more solid ground.

## OLD HINDU METHOD

### INFERIOR PLANETS

The different items of calculation are arranged as follows —

Sun	Planet	Sheegra kendra	Sheegra ra lla	Manda ken tra	Man la sphuta	Sheegra ken tra	Sph uta
रवि	ग्रह	शीघ्रकेन्द्र	शीघ्रार्ध	मन्दकेन्द्र	मन्दस्फुट	शीघ्रकेन्द्र	स्फुट

The sun and planet are written respectively under the first two headings. Sun is subtracted from the planet when शीघ्रकेन्द्र is got. There are tables called शीघ्रज्या which are entered into according as the शीघ्रकेन्द्र is मकरादि (between 270 to 90) or कर्करादि (90 to 270). Half of this equator is added to or subtracted from Sun according as शीघ्रकेन्द्र is मेघादि (0—180) or तुलादि (180—360°). This is called शीघ्रार्ध. This is subtracted from the Apse of the planet when Manda kendra (मन्दकेन्द्र) is got. There is a table as मन्दज्या. This is entered into and the effect thereof is applied to sun when मन्दस्फुट is got. This is again subtracted from the planet when the शीघ्रकेन्द्र is got. As before the शीघ्रज्या is entered into and the effect thereof is applied to the मन्दस्फुट. The result is the true longitude of the planet.

### SUPERIOR PLANETS.

In this the order of the items is as follows

Planet	Sun	Manda ken tra	Mandar lla	Sheegra kendra	Sheegra ra lla	Manda kendra	Manda sphuta	Sheegra ken tra	Sph uta
ग्रह	रवि	मन्दकेन्द्र	मन्दार्ध	शीघ्रकेन्द्र	शीघ्रार्ध	मन्दकेन्द्र	मन्दस्फुट	शीघ्रकेन्द्र	स्फुट

In this मंदकेद्रं is got by subtracting the planet from the Apse. Half of the effects got from the मंदज्या are applied to Planet we get मादधि. This is subtracted from sun when शीघ्रकेद्रं is got. The शीघ्रज्या (कस्य्यादि मरुगादि) is entered into and half the effects got there of are applied to मादधि we get शीघ्राधि. This is again subtracted from Apse when we get the ॥ Manda-kendra मंदकेद्रं. Again the मंदज्या फलं is got and applied to the original planet when we get मंदस्फुट. This is subtracted from Sun when ॥ शीघ्रकेद्रं is got. The शीघ्रज्याफलं is got and applied to the मंदस्फुटं when the true longitude of the planet is got.

It would have been observed that in the case of the inferior planets the position of the planet in the Hel orbit is entirely ignored inasmuch as none of the effects are applied to it. The mutual distances of the planet earth and sun are not taken into consideration at all and it is needless to say that the longitudes worked out according to the old Hindu tables will not be true to observation.

Further the eccentricities being sufficiently high in some cases the higher powers of  $e$ , should not have been ignored as appears from the old Hindu tables.

## RECTIFICATION

The following revised tables are herewith given which could be used while working according to the instructions hereunder.

The order of the terms for all planets are —

Planet	Apse	Manda Kendra	Manda Sphuta	Ravi Sphuta	Sheegra Kendra	*Sphuta
ग्रह	मंदोच्च	मंदकेद्र	मंदस्फुट	स्फुटगति	शीघ्रकेद्र	स्फुट

Subtract planet from Apse, when the मंदकेद्र is got. Enter the Mandajya मंदज्या. While using Mandajya when the मंदकेद्र is मेपादि, upto 90° use the top line and above that till 180 the lower one with the भुजं or sine argument when तुलादि, reverse the operation but taking care to take the भुजं in each case before entering the table.

Apply the effect got additive or subtractive to the planet according as the मंदकेद्र is मेपादि or तुलादि. We get मंदस्फुट; Subtract it from sun's



The Ptolemaic Theory stated that the Earth was at the centre of the universe and that the sun and other planets are performing their ceaseless journey about the Earth. This had the consent of the ancient mythologies of the world and the force of religious authorities with their usual oppression was so much that they could not even entertain any theory which was otherwise.

This theory could not explain the theory of Retrograde motion and stationary positions of planets and such other things as easily or rationally as the present Copernican theory does.

Thanks to the Copernican theory the Kepler's Laws and later on Newton's discoveries a great headway has been made in the field and things have been established on a more solid ground.

## OLD HINDU METHOD

### INFERIOR PLANETS

The different items of calculation are arranged as follows —

Sun	Planet	Sheegra kendra	Sheegra ra lha	Manda kendra	Man la sphuta	Sheegra kendra	Sphuta
रवि	ग्रह	शीघ्रकेंद्र	शीघ्रार्ध	मंदकेंद्र	मदस्फुट	शीघ्रकेंद्र	स्फुट

The sun and planet are written respectively under the first two headings. Sun is subtracted from the planet when शीघ्रकेंद्र is got. There are tables called शीघ्रज्या which are entered into according as the शीघ्रकेंद्र is मकरादि (between 270 to 90) or कर्करादि (90 to 270). Half of this equation is added to or subtracted from Sun according as शीघ्रकेंद्र is मेघादि (0°—180°) or तुलादि (180—360°). This is called शीघ्रार्ध. This is subtracted from the Apse of the planet when Manda kendra (मंदकेंद्र) is got. There is a table as मदज्या. This is entered into and the effect thereof is applied to sun when मदस्फुट is got. This is again subtracted from the planet when the शीघ्रकेंद्र is got. As before the शीघ्रज्या is entered into and the effect thereof is applied to the मदस्फुट. The result is the true longitude of the planet.

### SUPERIOR PLANETS.

In this the order of the items is as follows

Planet	Sun	Ma lla kendra	Man lar- lha	Sheegra kendra	Sheegra ra lha	Manda kendra	Manda sphuta	Sheegra kendra
ग्रह	रवि	मंदकेंद्र	मादार्ध	शीघ्रकेंद्र	शीघ्रार्ध	मंदकेंद्र	मदस्फुट	शी

A rough way of the determination of the  $\hat{\text{शीघ्रफलं}}$  to be applied, is got by using the multipliers,

$$\frac{\sqrt{1+2.31+3.01 \cos (\hat{\text{शीघ्रकेंद्र}})}}{\sqrt{1+\frac{72.347}{\text{Hel. Vel}}+\frac{17.01}{\sqrt{\text{Hel. Vel}}} \cos (\hat{\text{शीघ्रकेंद्र}})}} \quad (\text{for Mars})$$

$$\frac{15.67}{\sqrt{\text{Hel. Vel}}} \quad (\text{for Mercury}) \quad \frac{\sqrt{\text{Hel. Vel}}}{2.2323} \quad (\text{for Jupiter}),$$

$$\frac{9.8}{\sqrt{\text{Hel. Vel}}} \quad (\text{for Venus}) \quad \text{and} \quad \frac{\sqrt{\text{Hel. Vel}}}{1.4142} \quad (\text{for Saturn})$$

to the  $\hat{\text{शीघ्रफलं}}$  already found out from the tables. This rectified  $\hat{\text{शीघ्रफलं}}$  is applied to the  $\hat{\text{मंदस्फुट}}$  in the case of superior planets and to  $\hat{\text{स्फुटगति}}$  for the rest. The result will be the correct longitude.

## GEOCENTRIC VELOCITY.

The difference in the Heliocentric Velocities of the planet and sun will be the velocity of  $\hat{\text{शीघ्रकेंद्र}}$ . The effect of  $\hat{\text{शीघ्रफलं}}$  for this difference, considered between the items where the sine argument of the  $\hat{\text{शीघ्रकेंद्र}}$  falls is got and multiplied by the multiplier. The rectified correction is applied positive or negative according as the  $\hat{\text{शीघ्रकेंद्र}}$  is  $\hat{\text{मकरादि}}$  or  $\hat{\text{वृश्चिकादि}}$ , to the Hel. Vel. of Earth, in the case of Mercury and Venus and to the Hel. velocity of the planet, in the case of Mars, Jupiter and Saturn.

(Note— The tables that are given here differ from those referred to in the Hindu old method especially in the  $\hat{\text{मंदस्फुट}}$  of the inferior planets.)

## TABLE OF MARS.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	60	120	179	237	292	345	396	450	496	545	590	634	677	719	760	800
0	77	151	223	292	357	417	470	517	557	599	643	682	719	755	790	825
$\frac{1}{2}^\circ$	143	287	429	567	707	843	975	1103	1225	1342	1454	1561	1663	1761	1855	1945
$\frac{1}{4}^\circ$	125	249	371	491	607	720	829	935	1037	1135	1229	1319	1405	1487	1565	1639

longitude (रविस्फुट). We get शीघ्रकेंद्र. Enter शीघ्रज्या कन्यादि or मकरादि. Find the effect and note it down. In the case of inferior planets subtract स्फुटरवि from the मंदस्फुट, as the Hel velocities of inferior planets are always greater than that of sun, to get शीघ्रकेंद्र.

## HELIOCENTRIC VELOCITY (मंदस्फुटगति)

Multiply the daily mean velocity of the planet in mts by the difference in the items wherein the मंदकेंद्रमुज (sine argument of the mean anomaly) falls, and divide by 360. The quotient will be the correction to be applied to the mean velocity + or - according as the मंदकेंद्र is कन्यादि or मकरादि. The result is the True Heliocentric Velocity (Even if in कन्यादि, it be a case of decreasing items, the effect shall be taken negative)

The शीघ्रफलं got from the tables with the शीघ्रकेंद्र has to be applied with the following multipliers. Find the sine of the शीघ्रफलं with the proper sign. Apply the multipliers to the sine. Find whose sine is the product. That angle will be the rectified शीघ्रफलं or स्फुटीकृतशीघ्रफलं.

Mars	$\sqrt{3.31+3.04 \cos \theta} / \sqrt{1+1.2273 \frac{v}{v'} + 2.214 \sqrt{\frac{v}{v'}} \cos \theta}$
Mercury	$\sqrt{6.706+4.776 \cos \theta} / \sqrt{1+1.3741 \frac{v}{v'} + 2.344 \sqrt{\frac{v}{v'}} \cos \theta}$
Jupiter	$\sqrt{27.97+10.51 \cos \theta} / \sqrt{1+2.2803 \frac{v}{v'} + 3.02 \sqrt{\frac{v}{v'}} \cos \theta}$
Venus	$\sqrt{2.910+2.765 \cos \theta} / \sqrt{1+1.1749 \frac{v}{v'} + 2.168 \sqrt{\frac{v}{v'}} \cos \theta}$
Saturn	$\sqrt{91.94+19.07 \cos \theta} / \sqrt{1+3.0761 \frac{v}{v'} + 3.508 \sqrt{\frac{v}{v'}} \cos \theta}$

Where  $\theta$  is the शीघ्रकेंद्र of the planet  $v$  and  $v'$  are the Heliocentric Velocities of the Earth and planet

If taken without these multipliers, the शीघ्रफलं merely got from the tables will not be accurate and that is how the ordinary Hindu almanacs give planetary positions which are at variance with those of the more correctly <sup>correct</sup> ~~osar~~ western almanacs

A rough way of the determination of the शीघ्रफलं to be applied, is got by using the multipliers,

$$\sqrt{1 + 2.31 + 3.04 \cos (\text{शीघ्रकेंद्र})} \quad (\text{for Mars})$$

$$\sqrt{1 + \frac{72.547}{\text{Hel. Vel.}} + \frac{17.04}{\sqrt{\text{Hel. Vel.}}} \cos (\text{शीघ्रकेंद्र})}$$

$$\frac{15.67}{\sqrt{\text{Hel. Vel.}}} \quad (\text{for Mercury}), \quad \frac{\sqrt{\text{Hel. Vel.}}}{2.2323} \quad (\text{for Jupiter}),$$

$$\frac{9.8}{\sqrt{\text{Hel. Vel.}}} \quad (\text{for Venus}) \text{ and } \frac{\sqrt{\text{Hel. Vel.}}}{1.4142} \quad (\text{for Saturn})$$

to the शीघ्रफलं already found out from the tables. This rectified शीघ्रफलं is applied to the मंदस्फुट in the case of superior planets and to स्फुटरवि for the rest. The result will be the correct longitude.

## GEOCENTRIC VELOCITY.

The difference in the Heliocentric Velocities of the planet and sun, will be the velocity of शीघ्रकेंद्र. The effect of शीघ्रफलं for this difference, considered between the items where the sine argument of the शीघ्रकेंद्र falls is got and multiplied by the multiplier. The rectified correction is applied positive or negative according as the शीघ्रकेंद्र is मकरादि or कर्करादि, to the Hel. Vel. of Earth, in the case of Mercury and Venus and to the Hel. velocity of the planet, in the case of Mars, Jupiter and Saturn.

(Note— The tables that are given here differ from those referred to in the Hindu old method especially in the मंदज्या of the inferior planets)

## TABLE OF MARS.

Items	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
मंदज्या	0	60	120	179	237	292	345	396	450	486	525	559	593	612	629	640
	0	76	151	223	292	357	417	470	517	557	590	615	632	642	645	640
मकरादि	0	143	245	426	566	707	843	978	1113	1245	1372	1492	1622	1732	1855	1953
कर्करादि	0	620	1252	1697	2031	2201	2322	2392	2414	2401	2364	2308	2236	2151	2059	1974

## TABLE OF MERCURY.

Items	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
मंदज्या	0	120	238	355	469	580	686	789	886	977	1063	1142	1214	1277	1331	1374
मकरादि	0	191	381	565	735	890	1027	1144	1241	1318	1373	1409	1424	1423	1406	1374
कक्ष्यादि	0	101	199	290	390	483	561	660	744	821	900	971	1037	1097	1150	1197
कक्ष्यादि	0	223	435	621	791	931	1044	1134	1200	1245	1274	1288	1276	1262	1233	1197

## TABLE OF JUPITER.

Items	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
मंदज्या	0	33	65	97	127	157	185	211	236	258	277	291	309	319	326	330
मकरादि	0	37	73	109	142	174	204	231	256	277	295	309	320	327	330	330
कक्ष्यादि	0	59	119	176	231	281	325	381	429	472	511	546	576	601	621	637
कक्ष्यादि	0	89	175	250	330	398	458	510	553	589	615	631	647	649	616	637

## TABLE OF VENUS.

Items	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
मंदज्या	0	5	10	15	19	23	27	31	35	38	41	43	45	46	47	47
मकरादि	0	5	10	15	19	23	27	31	35	38	41	43	45	46	47	47
कक्ष्यादि	0	152	303	454	604	753	901	1050	1197	1340	1483	1623	1761	1893	2022	2150
कक्ष्यादि	0	927	1657	2162	2466	2645	2753	2783	2788	2734	2667	2579	2483	2382	2269	2150

## TABLE OF SATURN.

Items	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
मंदज्या	0	38	75	111	147	181	214	244	272	298	321	341	357	370	379	383
मकरादि	0	43	86	127	167	204	239	271	299	324	344	361	373	381	384	283
कक्ष्यादि	0	36	71	105	138	170	199	228	253	277	297	317	333	345	354	362
कक्ष्यादि	0	45	79	130	170	207	239	270	295	316	332	347	356	362	363	362

Each item in all these tables stands for 6° of the sine argument of the respective functions (केन्द्र).

## EXAMPLE

Mean longitude of Mars	5—0—30—53
Apse of Mars	4—11—49—37
Suns true longiturd	2—29—21—22

भौम	मंदोच्च	मंदकेंद्र	मंदस्फुट	स्फुटरवि	शीघ्रकेंद्र	स्फुट
5	4	11	4	2	10	4
0	11	11	27	29	1	6
30	49	18	25	21	56	6
53	37	44	14	22	8	45
—	—	—	—	—	—	—
31			26	57	30	
26			23	13	50	

मंदकेंद्र is got by subtracting the planet from its apse. This is more than 180° and past 270° also. The item which began from 0 item of the lower line has come to the top line. The item will be found from the sine argt. It is  $12^s - (11^s - 11' - 18'' - 44'') = 18^s - 41' - 16''$ . As 6° corresponds to 1 item the no. of past items is 3 and  $41' - 16''$  remains. The figure for the 3rd item is 179 and difference for the next is + 58. ∴ Proportional for  $41' - 16'' = \frac{41 - 16 \times 58}{160} = 6' - 39''$

∴ Equation for  $18^s - 41' - 16'' = 179^s - 0' plus 6' - 39'' = 185^s - 39' - 3'' = 3^s - 5' - 39''$

As the मंदकेंद्र is more than 180°, this is negative. Hence subtracting it from the planet we get  $4^s - 27' - 25' - 14''$ . The difference between the items wherein the sine argument of the मंदकेंद्र lies is + 58. Find  $31' - 26'' \times \frac{58}{360}$  it is  $5' - 3''$ . As the मंदकेंद्र is मकरादि, this has to be subtracted from the mean velocity of Mars  $31' - 26''$  when we get the true heliocentric velocity of Mars as  $26' - 23''$ .

Then स्फुटरवि is written. This less the मंदस्फुट will give the शीघ्रकेंद्र. It is  $10^s - 1' - 56' - 8''$ . As this is more than 270° we have to refer to the मकरादिज्या, with its sine argument. The sine argument is  $1^s - 28' - 3'' - 52'' = 58^s - 3' - 52''$ . This will be 9 items plus  $4^s - 3' - 52''$ . From the मकरादि

tables we get  $\left(1215 + \frac{243-52 \times 127}{360}\right)$

which is equal to  $1331' - 2'' = 22^\circ - 11' - 2''$ . This is minus as शीघ्रचंद्र is more than  $180^\circ$ .

This sine of this is  $-2775804$

The multiplier is  $\frac{\sqrt{3 \ 31 + 3 \ 04 \cos \theta}}{\sqrt{1 + 1 \ 2272 \times 2 \ 1695 + 2 \ 214 \times 1 \ 473 \cos \theta}}$

$$\left[ \frac{v}{v'} = \frac{57 \ 22}{26 \ 38} = 2 \ 1695 \text{ and } \sqrt{\frac{v}{v'}} = 1 \ 473 \right]$$

In this case  $\theta = 301^\circ - 56' - 8''$  and  $\cos \theta = .5299$

$\therefore$  The multiplier becomes  $\frac{\sqrt{3 \ 31 + 3 \ 04 \times .5299}}{\sqrt{3 \ 6623 + 3 \ 114 \times .5299}}$

$$= \sqrt{\frac{4 \ 9209}{6 \ 3124}} = .9621$$

Multiplied sine (शीघ्रफल)  $= -3775804 \times .9621$

$$= -3633834$$

This is sine of  $-(21^\circ - 18' - 29'')$

Applying this to the मंदस्फुट, we get  $4^\circ - 6' - 6'' - 15''$  This is the geocentric longitude

## GEOCENTRIC VELOCITY

The difference in the 9th and 10th items wherein the sine argument lies is  $+127$ . The difference in the Hel velocities of Planet and Earth is  $36' - 50''$ . Find  $\frac{30' - 50'' \times 127}{360} = 10' - 52''$

As the शीघ्रचंद्र is मकरादि; The effect on the velocity is positive and negative if it had been कन्यादि. (In कन्यादि also if the difference between the items is negative then the effect on velocity will be positive. This should be noted)  $\therefore$  The effect is  $+10' - 52''$ . Now apply the multiplier .9624 we get  $+10' - 36''$ . This is has to be added to the Hel vel of Mars. We get  $(26' - 23'') + (10' - 36'') = 36' - 59''$ . This is the Geocentric Velocity

The other planetary positions also could be calculated accordingly.

This chapter is specially intended for those who would make use of the Hindu tables as rectified here with as little use of Trigonometry as possible

# Chapter XXIII.

## APPENDIX

### TABLE OF TRIGONOMETRICAL RATIOS.

Deg	Sines	Common difference	Tangent	Common difference	Deg
0	.0000000	174524	.0000000	174551	90
1	.0174524	174471	.0174551	174657	89
2	.0348995	174365	.0349208	174870	88
3	.0523360	174205	.0524078	175190	87
4	.0697565	173992	.0699268	175619	86
5	.0871557	173725	.0874887	176155	85
6	.1045285	173408	.1051042	176804	84
7	.1218693	173038	.1227846	177562	83
8	.1391731	172614	.1405408	178436	82
9	.1564345	171937	.1583844	179426	81
10	.1736482	171608	.1763270	180533	80
11	.1908090	171027	.1943803	181763	79
12	.2079117	170394	.2125866	183116	78
13	.2249511	169708	.2308682	184598	77
14	.2419219	168971	.2493280	186212	76
15	.2588190	168184	.2679492	187962	75
16	.2756374	167343	.2867454	189953	74
17	.2923717	166453	.3057307	191890	73
18	.3090170	165512	.3249197	194079	72
19	.3255682	164519	.3443276	196426	71
20	.3420201	163473	.3639702	198938	70
21	.3583679	162387	.3838640	201622	69
22	.3746066	161245	.4040262	204486	68
23	.3907311	160055	.4244748	207539	67
24	.4067366	158817	.4452287	210790	66
25	.4226183	157528	.4663077	214249	65
26	.4383711	156194	.4877326	217928	64
27	.4539905	154811	.5095254	221840	63
28	.4694716	153380	.5317094	225997	62
29	.4848096	151904	.5543091	230412	61
30	.5000000	150381	.5773503	235103	60
31	.5150381	148812	.6008606	240088	59
32	.5299193	147197	.6248694	245382	58



33	5116390	145539	6494076	251009	57
34	5591929	143835	6745085	256990	56
35	5735764	142039	7002075	263350	55
36	5877853	140297	7265425	270116	54
37	6018150	138465	7535541	276715	53
38	6156615	136589	7812856	284984	52
39	6293204	134672	8097840	298156	51
40	6427876	132714	8390996	301871	50
41	6560590	130716	8692867	311173	49
42	6691306	128678	9004040	321111	48
43	6819981	126600	9325151	331737	47
44	6946584	124484	9656888	343112	46
45	7071068	122330	1 0000000	355303	45
46	7193398	120139	1 0355303	368384	44
47	7313537	117911	1 0723687	382433	43
48	7431448	115648	1 1106125	397441	42
49	7547096	113348	1 1503684	413852	41
50	7660444	111016	1 1917536	431436	40
51	7771460	108648	1 2348972	450444	39
52	7880108	106247	1 2799416	471032	38
53	7986355	103815	1 3270448	493371	37
54	8090170	101350	1 3763819	517661	36
55	8191520	98856	1 4281480	544130	35
56	8290376	96330	1 4825610	573040	34
57	8386706	93775	1 5398650	604695	33
58	8480481	91192	1 6003345	639450	32
59	8571673	88581	1 6642795	677713	31
60	8660254	85863	1 7320508	719970	30
61	8746197	83279	1 8040478	766787	29
62	8829476	80589	1 8807265	818840	28
63	8910065	77875	1 9626105	876933	27
64	8987940	75138	2 0503038	942031	26
65	9063078	72377	2 1445069	1015299	25
66	9135455	69594	2 2460368	1098156	24
67	9205049	66790	2 3558524	1192345	23
68	9271839	63965	2 4750869	1300022	22
69	9335804	61122	2 6050891	1423883	21
70	9396926	58260	2 7474774	1567385	20
71	9455186	55379	2 9042109	1734726	19
72	9510565	53433	3 0776835	1931691	18
73	9563048	49569	3 2708526	2165618	17
74	9612617	46641	3 4874144	2446364	16
75	9659258	43699	3 7320508	2787301	15
76	9702957	40744	4 0107809	3206950	14

77	9748701	37775	4 3814759	3781542	18
78	9781476	84796	4 7046301	4899239	12
79	9816272	81806	5 1445540	5267278	11
80	9848078	28807	5 6712818	6424697	10
81	9876883	25798	6 8187515	8016182	9
82	9902681	22781	7 1153697	10289767	8
83	9925462	19657	8 1443464	13700181	7
84	9945219	16728	9 5148645	19156875	6
85	9961947	13694	11 130052	2870614	5
86	9975641	10654	14 300666	4780471	4
87	9986295	7618	19 081187	9555116	3
88	9993908	4569	28 686258	28657709	2
89	9998477	1523	57 289962	Infinite	1
90	1 0000000	0	$\infty$		0
cosines			cotangents		

As the change of the tangent ratio from  $89^\circ$  to  $90^\circ$  is very rapid the following table for each minute from  $89^\circ$  to  $90^\circ$  is given as hereunder

89°—1'	58 261174	18	81 847041	35'	137 50745	52	429 71757
2	59 265872	19	83 813507	36	143 23712	53	491 10600
3	60 305820	20	85 939791	37	149 46502	54	572 95721
4	61 382905	21	88 143572	38	156 25908	55	687 54887
5	62 499154	22	90 463336	39	163 70019	56	859 43630
6	63 655741	23	92 908487	40	171 88540	57	1145 9153
7	64 858008	24	95 489475	41	180 93220	58	1718 8732
8	66 105473	25	99 217943	42	190 98419	59	3437 7467
9	67 401854	26	101 106900	43	202 21875	90°	Infinite
10	68 750087	27	104 17094	44	214 85762		
11	70 153346	28	107 42648	45	229 18166		
12	71 615070	29	110 89205	46	245 55198		
13	73 138990	30	114 58865	47	264 44080		
14	74 729165	31	118 54018	48	286 47773		
15	76 390009	32	122 77396	49	312 52137		
16	78 126342	33	127 32134	50	343 77371		
17	79 943430	34	132 21851	51	381 97099		

## TABLE OF CHARAPHALAM FOR DIFFERENT LATITUDES.

(Ascensional difference)

In Chapter IX on the position of the observer etc the use of Charaphalam has been explained with a table for Tanjore latitude  $10^\circ-47'N$

A note has also been given indicating how the चरं for different latitudes could be obtained by a small calculation. As some of the readers may find it slightly difficult, I am herewith appending a table of चरफले for each degree of latitude and for every 30" of the sine argument of tropical longitude. With the help of this a table of चरफले for each degree of the sine argument of the tropical longitude can be prepared and had for any desired latitude.

## AN EXAMPLE WILL MAKE THINGS CLEARER.

*Question*.—Required चरफले for 27° of latitude corresponding to 25° 47' 69" of sine argument of the tropical longitude —

चरफले for 25° 47' and 69" for latitude of Tanjore are 112'—197'—10" and 264'—33". The figures for 30", 60", 90" of sine argument of tropical longitude for Tanjore are 133'—1", 240'—40" and 284'—27" and those for the given latitude are 356'—22", 647'—8" and 766'—48" from the table.  $\therefore$  चरफले for 25° of the sine argument of the tropical longitude at the given latitude =  $\frac{112'-197'-10" \times 356'-22"}{133'-1"} = 302'-2"$

चरफले for 47" of the same

$$= 356'-22" + \left( \frac{197'-10" - 133'-1"}{107'-39"} \right) (290'-46")$$

(where 290'—46" = 647'—8" — 356'—22")

$$= 356'-22" + 173'-15"$$

and 107'—39" = 240'—40" — 133'—1"

$$= 529'-37"$$

—133'—1"

चरफले for 69" of the same

$$= 647'-8" + \frac{(264'-33" - 240'-40")}{(284'-27" - 240'-40")} (766'-48" - 647'-8")$$

$$= 647'-8" + 59'-48" = \underline{706'-56"}.$$

Argument — Sine argument of tropical longitude and latitude of place

Degree of titude	30° ' "	60° ' "	90° ' "	Degree of Latitude	30° ' "	60° ' "	90° ' "
0	0 0	0 0	0 0	34	472 23	860 35	1021 40
1	12 11	22 2	26 3	35	490 30	894 8	1061 51
2	24 23	44 5	52 7	36	509 5	928 34	1103 11
3	36 35	66 10	78 14	37	528 10	964 0	1145 46
4	48 48	88 18	104 23	38	547 46	1000 30	1189 37
5	61 5	110 29	130 37	39	567 55	1038 7	1234 58
6	73 23	132 44	156 55	40	588 41	1076 58	1281 50
7	85 44	155 5	183 20	41	610 5	1117 6	1330 21
8	98 8	177 31	209 53	42	632 10	1158 39	1380 39
9	110 35	200 5	236 34	43	654 59	1202 42	1432 53
10	123 8	222 48	263 25	44	678 35	1246 28	1487 14
11	135 44	245 38	290 27	45	703 2	1292 50	1543 51
12	148 26	268 38	317 41	46	728 23	1341 12	1602 58
13	161 14	291 50	345 9	47	754 43	1391 39	1664 48
14	174 8	315 14	372 51	48	782 5	1444 22	1729 37
15	187 10	338 51	400 49	49	810 37	1499 32	1797 44
16	200 18	362 43	429 4	50	840 21	1557 25	1869 27
17	213 35	386 49	457 39	51	871 26	1618 16	1945 13
18	227 0	411 13	486 34	52	903 58	1682 23	2025 17
19	240 35	435 55	515 50	53	938 4	1750 8	2110 47
20	254 20	460 55	545 29	54	973 53	1821 53	2201 40
21	268 16	486 17	575 36	55	1011 35	1898 8	2299 4
22	282 23	512 0	606 8	56	1051 20	1979 26	2403 51
23	296 48	538 8	637 10	57	1093 23	2066 25	2517 12
24	311 16	564 40	668 42	58	1137 55	2159 50	2640 34
25	326 2	591 40	700 48	59	1185 14	2260 45	2775 52
26	341 4	619 8	733 28	60	1235 39	2370 14	2925 38
27	356 22	647 8	766 48	61	1289 30	2489 47	3093 22
28	371 56	675 40	800 47	62	1347 14	2621 18	3284 12
29	387 48	704 47	835 29	63	1409 19	2767 16	3506 10
30	404 0	734 32	870 58	64	1476 22	2931 8	3773 22
31	420 32	764 56	907 16	65	1549 3	3117 47	4115 41
32	437 26	796 2	944 26	66	1628 15	3334 43	4631 20
33	454 42	827 55	982 27	66°	1674 13	3468 59	5400 0
				32			
				30°			

## TO FIND THE LOCAL SUNRISE AND SUNSET TIME

This is arrived at from a knowledge of चरं (ascensional difference) of the place प्राणं (equation of time due to obliquity) and मरफलं (equation of time due to eccentricity)

The  $\text{प्राणं}$  and  $\text{मंदफलं}$  are found and their net result is divided by 6 when the quotient will be  $\text{vighatikas}$ . This is added to or subtracted from 15  $\text{ghatikas}$  when the apparent noon is got. This converted to hours will give the Apparent Noon time. Next the  $\text{चरं}$  is added to or subtracted from 15<sup>h</sup>—5  $\text{vighatikas}$  after being converted similarly to  $\text{vighatikas}$  by dividing by 6 according as the tropical longitude of sun is less or greater than 180°. This will give the duration of half the day. This is added to the noon time one side and subtracted from the true noon on the other when the sunset and sunrise in  $\text{ghatikas}$  is got. These merely converted into hours will give the local time sunset and sunrise.

### EXAMPLE

On page 72, we have

$$\text{प्राणं} = 106' - 5'' +$$

$$\text{and मंदफलं} = 21' - 22'' -$$

Net result

$$= 84' - 43'' +$$

This converted to  $\text{vighatikas}$

$$= 14' 12'' +$$

Being +, this has to be added to 15  $\text{ghatikas}$

We get local apparent noon

$$= 15^{\text{gh}} - 14' 12'' \text{vighat}$$

$$= 12^{\text{h}} - 5^{\text{m}} - 39^{\text{sec}}$$

$\text{चरं}$  is  $311' - 36'' (-)$  for planetary correction but reversedly for duration of day. Converted to  $\text{vighatikas}$  we get  $51' 03'' +$ . Adding this to  $15^{\text{gh}} - 5' \text{vighats}$ , we get  $15^{\text{gh}} - 56' 03'' \text{vighat}$  or  $6^{\text{hrs}} - 22^{\text{mts}} - 46^{\text{sec}}$

Local sunrise time

$$= \begin{array}{r} \text{h} \quad \text{m} \quad \text{sec} \\ 12 - 5 - 39 - \end{array}$$

$$6 - 22 - 46$$

$$\hline 5 - 42 - 53 \text{ A M}$$

and Local sunset time

$$= \begin{array}{r} 12 - 5 - 39 + \end{array}$$

$$6 - 22 - 46$$

$$\hline 6 - 28 - 25 \text{ P M}$$

### DECLINATION.

In the chapter on declinations we have mentioned them to be + or — according as they are North or South. Theoretically Astronomically

is correct but in the determination of the strength of a planet according to Hindu Astrology there is a special consideration for the north and south directions which could be had on a reference to any authoritative book on the subject. Hence these need not be used for those purposes with the directions assigned here which are quite correct for purpose of any Astro-mical transformation of coordinates etc

## CONCLUSION AND RETROSPECTIVE.

Now we have come to the end of the little treatise bringing out for the information of the learned readers the details of Hindu Astronomy and European Astronomy striking parallels here and there and giving them an opportunity to have a sound working knowledge of the principles laid therein. I hope the learned readers will encourage me in my literary pursuits and co-operate with me in making the work a thorough success

Dedicated to the revered feet of my Guru

Tonpai Joshi Raghavendrachariar of Tanjore

